Math 4680/5680, Topics in Logic and Computation, Winter 2017

Handout 1: Problems for propositional logic

9. $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$.

10. $A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$.

11. $A \wedge (A \vee B) \equiv A$.

12. $A \vee (A \wedge B) \equiv A$.

13. $\vdash \neg(A \wedge (\neg A))$.

14. $A \rightarrow (\neg A) \vdash \neg A$.

15. $(A \rightarrow B) \wedge (A \rightarrow C) \equiv A \rightarrow (B \wedge C)$.

16. $A \rightarrow B,\ B \rightarrow C \vdash A \rightarrow C$.

17. $\vdash A \rightarrow (B \rightarrow A)$.

18. $(A \rightarrow B) \rightarrow C \vdash B \rightarrow C$.

19. $A \rightarrow B \vdash (\neg B) \rightarrow (\neg A)$.

20. $A \rightarrow B \vdash \neg(A \wedge (\neg B))$.

21. $A \wedge (\neg B) \vdash \neg(A \wedge B)$.

22. $(\neg A) \rightarrow (\neg B) \vdash B \rightarrow (\neg(A \rightarrow (\neg B)))$.

23. $\vdash \neg(A \leftrightarrow (\neg A))$.

24. $A \vee B \vdash (B \rightarrow A) \rightarrow A$.

25. $A \vee B \vdash (\neg B) \rightarrow (C \rightarrow A)$.

26. $(B \rightarrow A) \wedge (A \vee B) \vdash A$.

27. $A \vee B \vdash (\neg A) \rightarrow B$.

28. $(\neg A) \vee B \vdash A \rightarrow B$.

29. $(\neg A) \vee (\neg B) \vdash \neg(A \wedge B)$.

30. $\neg(A \vee B) \equiv (\neg A) \wedge (\neg B)$.

31. $(A \vee B) \rightarrow C \equiv (A \rightarrow C) \wedge (B \rightarrow C)$.

32. $(A \rightarrow B) \vee (A \rightarrow C) \vdash A \rightarrow (B \vee C)$.

33. $(A \rightarrow C) \vee (B \rightarrow C) \vdash (A \wedge B) \rightarrow C$.

34. $((\neg A) \vee C) \wedge (B \rightarrow C) \vdash (A \rightarrow B) \rightarrow C$.

35. $\vdash (A \rightarrow C) \rightarrow ((B \rightarrow C) \rightarrow ((A \vee B) \rightarrow C))$.

36. $(A \rightarrow C) \vee (B \rightarrow D) \vdash (A \wedge B) \rightarrow (C \vee D)$.

37. $A \rightarrow B,\ C \rightarrow D,\ (\neg B) \vee (\neg D) \vdash (\neg A) \vee (\neg C)$.

38. $A \rightarrow (C \vee D),\ (A \vee D) \vee E,\ A \rightarrow (\neg C) \vdash D \vee E$.

39. $A \vdash \neg\neg A$.

40. $\neg\neg\neg A \equiv \neg A$.

(NOTE: The $\neg\neg$ rule is required for nos. 41-53.)

41.* $\vdash A \vee (\neg A)$.

42. $(A \rightarrow B) \vee C,\ A \rightarrow (\neg C) \vdash (B \rightarrow C) \rightarrow (\neg A)$.

43. $\neg(A \wedge (\neg B)) \vdash A \rightarrow B$.

44. $(\neg B) \rightarrow (\neg A) \vdash A \rightarrow B$.

45.* $A \rightarrow B \vdash (\neg A) \vee B$.

46.* $(B \rightarrow A) \rightarrow A \vdash A \vee B$.

47.* $(A \rightarrow B) \rightarrow C \vdash A \vee C$.

48.* $(\neg A) \rightarrow B \vdash A \vee ((\neg A) \wedge B)$.

49.* $\vdash (A \wedge B) \vee (\neg A) \vee (\neg B)$.

50.* $\neg(A \wedge B) \vdash (\neg A) \vee (\neg B)$.

51.* $\vdash (A \rightarrow B) \vee (B \rightarrow A)$

52.* $A \rightarrow (B \vee C) \vdash (A \rightarrow B) \vee (A \rightarrow C)$.

53.* $(A \wedge B) \rightarrow (C \vee D) \vdash (A \rightarrow C) \vee (B \rightarrow D)$. (cf.no.36)

54. Prove that if $\varphi \wedge \psi \vdash \theta$ and $\varphi \wedge \theta \vdash \zeta$ then $\varphi \wedge \psi \vdash \zeta$.

55. Prove that if $\varphi \vdash \psi$ then

(i) $\varphi \wedge \theta \vdash \psi \wedge \theta$, (ii) $\varphi \vee \theta \vdash \psi \vee \theta$,

(iii) $\theta \rightarrow \varphi \vdash \theta \rightarrow \psi$, (iv) $\psi \rightarrow \theta \vdash \varphi \rightarrow \theta$.

56. Prove that $\Gamma \cup \{\varphi\} \vdash \psi$ iff $\Gamma \vdash \varphi \rightarrow \psi$ (Γ is any set of formulae).