Foundational Methods in Computer Science 2012
Dalhousie University, Halifax, Canada
June 14–17, 2012

Abstracts

Subashis Chakraborty (Calgary): "The proof theory of message passing with protocols"

Abstract: Sequential programs are connected through the λ-calculus by the Curry-Howard-Lambek isomorphism to intuitionistic proofs and cartesian closed category. The Curry-Howard isomorphism allows proofs to be viewed as programs and propositions as types. But what is the proof theory of concurrent programs?

Message passing is a key element of concurrent programming. In the paper "The Logic of Message Passing" (2007) Cockett and Pastro showed how message passing could be accommodated in the proof theoretic framework for concurrency. They used the two-sided proof theory of cut elimination and described a categorical semantics of the system.

In this talk, we will describe this two-tier message passing logic and its categorical semantics. We will focus on providing an intuition for the system. The cut elimination of the system provides an operational semantics, which we shall illustrate with examples.

From the programming perspective, in the both levels (message and message passing) of this system, datatypes are desirable but at the message passing level datatypes are of significant interest as they allow the expression of communication protocols.The talk will illustrate how protocols may be used.

Robin Cockett (Calgary): "Can you differentiate a polynomial?"

Abstract: Everyone (probably) knows how to differentiate an ordinary polynomial. But how good are you at differentiating a polynomial functor? By the end of the talk everyone, should be able to differentiate a tree ...

The differentiation of combinatorial species and the possibility of differentiating polynomial functors was, in fact, one of the original motivations for developing the notion of a Cartesian differential category. The talk, thus, revisits these motivational examples and takes a looks at the question of whether polynomial functors form a differential category. Given the recent efforts to provide a clean formulation for polynomial functions by Gambino, Kock, Weber, and many others, it seems particularly appropriate to revisit these questions as they now seem answerable.

Geoff Cruttwell (Ottawa): "Differential structure, tangent structure, and SDG"

Abstract: In this talk, we explore how Rosicky's notion of tangent structure (an axiomatization of the tangent bundle functor on smooth manifolds) relates to Cartesian differential categories and synthetic differential geometry (SDG). On the one hand, every instance of tangent structure contains a subcategory which is Cartesian differential; as a result, categories with a differential form a coreflective subcategory of categories with a tangent functor. On the other hand, we expand upon a result of Rosicky's, showing that every instance of representable tangent structure generates a model of SDG.

Jeff Egger (Dalhousie): "A categorical look at Fourier transforms"

Abstract: For a locally compact group G, the convolution algebra of integrable functions G–>C, denoted L^1(G), carries some of the structure of a dagger Hopf monoid in the category of Banach spaces and linear contractions—in general, it lacks both a unit and a comultiplication. When G is abelian, then the C*-algebra associated to (the underlying space of) its Pontryagin dual, denoted C_0(G^), carries the same amount of structure; moreover, the Fourier transform Phi_G : L^1(G) -> C_0(G^) preserves it. But Phi does not define a natural transformation out of the whole category of locally compact groups: it is necessary to restrict to the lluf subcategory of *open* continuous homomorphisms. This is because a continuous homomorphism is measure-zero-reflecting (wrt Segalified Haar measure) iff it is open, and in the abelian case this is equivalent to its dual being proper.

Jonathan Gallagher (Calgary): "Differential combinatory algebras"

Abstract: The differential lambda calculus (DLC) extends the lambda calculus with structure that allows for the partial derivative of a lambda term to be taken. The DLC led to the study of differential Turing categories; these are an attempt to directly understand the models of computability that arise when every computable function has a computable derivative.

However, the only known examples of differential Turing categories arise as models of the untyped differential lambda calculus, and these models, while interesting, are total. One obvious way to generate a model with partiallity is to build a differential partial combinatory algebra (DPCA) based on a total differential combinatory algebra using the nontermination of its rewriting system to express partiallity.

This talk will focus on the road to defining such a DPCA; in particular, we will focus on differential combinatory algebras. This path shed light on some interesting equational models and rewriting theory. We will investigate these equational models and show that we obtain some new cartesian differential categories. We will also investigate the rewriting systems associated with these equational models and their properties: most importantly, confluence modulo an equivalence relation.

Brett Giles (Calgary): “Linear Quantum Programming Language (LPQL) and implementing a Quantum Stack”

Abstract: LQPL (Linear Quantum Programming Language). LQPL was inspired by Peter Selinger’s paper "Towards a Quantum Programming Language", which gives a categorical semantics for quantum computing. However, it adds several interesting features.

LQPL programs are compiled to quantum stack code. The LQPL compiler enforces linearity requirements, performs type inference, and ensures a balanced quantum stack is maintained for measures and case statements. The stack code is evaluated using a quantum stack emulator. To evaluate recursions, it uses a lazy infinite list of approximations.

LPQL adds a direct representation of quantum control, the ability to use classical data, and classically controlled algebraic data types (such as lists). The talk will focus on illustrating these features and explaining how they are implemented.

Transforms and measures are performed on the quantum stack by modifying stack "locations" in place. Quantum control, however, is implemented by rotating the controlling qubits to the top of the stack and then applying operations conditionally below those controlling qubits. This and the implementation of recursion appear to be the two most expensive aspects of the quantum stack emulation.

The latest implementation of LPQL separates the compiler and the user interface from the quantum stack emulator. The objective of this effort is to facilitate optimizations and alternative implementations of the quantum stack emulator itself.

Willem Heijltjes (LIX): “Proof nets and semi-star-autonomous categories”

Abstract: I will present a notion of semi-star-autonomous category, such that proof nets for multiplicative linear logic without units form the free such category. The challenge is to capture the proof nets with a single formula in the conclusion, which are traditionally modelled by morphisms from the tensor unit 1 in a star-autonomous category, in the absence of this unit. Several proposals towards a solution were put forward in the middle of the past decade, by Robin Houston and Dominic Hughes, by Kosta Dosen, and by François Lamarche and Lutz Straßburger. The present work completes the latter of these. Single-formula proof nets are interpreted via a set-valued functor called the “virtual unit”, which takes the role of hom(1,-). Subject to additional coherence axioms, this functor is the central ingredient in this notion of semi-star-autonomous category. This is joint work with Lutz Straßburger.

Pieter Hofstra (Ottawa): "Isotropy and Crossed Toposes"

Abstract: Various mathematical structures, such as groupoids, inverse semigroups and etale groupoids, contain a substructure which is to be regarded as the isotropy of that structure. This isotropy substructure in fact forms an internal group object in the classifying topos.

In order to understand what is special about these groups, we will exhibit a monad on the category of grouped toposes and explain how the algebras are to be regarded as a crossed structure on a topos. In the special case where the topos is G-sets for a group G, this reduces to the usual notion of crossed G-module. In the motivating examples, there is a crossed structure associated to the isotropy group which is characterized by a universal property.

Just as ordinary crossed modules are equivalent to 2-groups, crossed toposes may also be regarded as certain categories internal to the category of toposes. The notion of a discrete fibration on such internal category turns out to be the key to the correct topos-theoretic generalization of the so-called Clifford-fundamental decomposition of an inverse semigroup.

(Joint work with Jonathon Funk and Benjamin Steinberg)

Peter LeFanu Lumsdaine (Dalhousie): "Fibrations as Types — a categorical road to intensional type theory"

Abstract: The subfield currently trading as Homotopy Type Theory arose, historically, out of proof-theoretic work on the implications of Martin-Löf's intensional type theory, which itself was formulated syntactically based on philosophical and computational considerations.

I will give an alternate history, via more traditional categorical logic, showing how fibrational models and the language of intensional type theory can be reached from semantic considerations, and how this leads to a "logic for homotopy theory"; and I will survey the state of the art in homotopy type theory from this point of view.

Ernie Manes (Massachusetts): "More work for Robin: Universal algebra in everyday programming logic, and concomitant challenges for restriction categories"

Abstract: A network is the sum of its paths, hence lives in an abelian monoid. An algebra of tests acts on these monoids. We consider three cases. Case I: tests are total. Case II: tests can diverge, but preserve computability. Case III: oracle for halting. Case I is Boolean algebra but, in case II (everyday programming logic), the "and" and "or" connectives are not commutative. In all three cases, the actions are modules over a suitable rig.

This is a tutorial. Relevant aspects of universal algebra such as Birkhoff's subdirect product theorem, primal algebras and Pixley's theorem will be introduced and discussed.

Presumably, the "Boolean restriction categories" of Cockett and Manes 2009 in MSCS provide a semantic framework for case I. While no nontrivial additive category can be a restriction category with restriction zeroes, a suitable additive completion exists allowing a general formulation of both cases I, II in universal-algebraic terms. Further research directions for restriction categories will be considered.

Francisco Marmolejo (UNAM): "No-iteration pseudomonads"

Abstract: The no-iteration presentation of monads (Kleisli triples) has a higher dimensional version for pseudomonads, and their algebras. We will take a look at the precise formulation for this no-iteration pseudomonads and its implications for distributive laws between pseudomonads.

Micah McCurdy (Dalhousie): "Introduction to Adiabatic Quantum Computing"

Abstract: Adiabatic quantum computing has recently emerged as a novel method of harnessing quantum effects for computation. I will give a brief introduction to the method, its mechanism, the state of the art implementations, and some of the progress of the Kyriakidis group at Dal in solving factoring problems.

Philip Mulry (Colgate): "An Overview of Some Categorical Approaches to Computation"

Abstract: On the occasion of Robin Cockett's 60th, this talk will consider some past work on categorical models of computation and place this work in the context of several present approaches. We will also highlight some of the challenges to the subject that still remain.

Susan Niefield (Union): "A double approach to variation for bicategories"

Abstract: For a small category B, it is well known that the 2-slice category Cat/B is equivalent to a 2-category $\mathrm{Lax}(B, \mathcal{S}pan)$ of lax functors $B \to \mathcal{S}pan$, which in turn is equivalent to the 2-category $\mathrm{Lax}_{\mathrm{N}}(B, \mathcal{P}rof)$ of normal lax functors $B \to \mathcal{P}rof$. In joint work (unpublished) with Cockett and Wood during the first half of the last decade, we generalized this equivalence by replacing B by a bicategory $\mathcal{B}$, and showing that $\mathrm{Lax}(\mathcal{B}, \mathcal{S}pan)$ is equivalent to a 2-category $\mathrm{LDF}/\mathcal{B}$ whose objects are are local discrete fibrations over $\mathcal{B}$. In addition, we showed that $\mathrm{LDF}/\mathcal{B}$ is equivalent to the 2-category $\widehat{\mathcal{B}}\text{-}\mathrm{Cat}$ of categories enriched in a related bicategory $\widehat{\mathcal{B}}$. We also obtained an equivalence $\mathrm{Lax}(\mathcal{B}, \mathcal{S}) \simeq \mathrm{Lax}_{\mathrm{N}}(\mathcal{B}, \mathrm{Mod}(\mathcal{S}))$, where $\mathrm{Mod}(\mathcal{S})$ is the bicategory whose objects are monads in a bicategory $\mathcal{S}$ and morphisms are modules. This generalizes the above mentioned equivalence involving $\mathcal{P}rof$, since $\mathrm{Mod}(\mathcal{S}pan) \simeq \mathcal{P}rof$.

For appropriate double categories $\mathbb{B}$ and $\mathbb{S}$, Paré recently introduced a double category $\mathbb{L}\mathrm{ax}(\mathbb{B}, \mathbb{S})$ whose objects are lax functors $\mathbb{B} \to \mathbb{S}$, and showed that the double slice $\mathbb{C}\mathrm{at}/\!\!/B$ is equivalent to $\mathbb{L}\mathrm{ax}(\mathbb{V}B, \mathbb{S}\mathrm{pan})$, where $\mathbb{V}B$ denotes the horizontally discrete double category whose vertical morphisms are those of B, thus generalizing $\mathrm{Cat}/B \simeq \mathrm{Lax}(B, \mathcal{S}pan)$ to the related double categories.

In this talk, we show that our 2-categories have vertical structure and the equivalences extend to the double categories that arise. In particular, $\mathbb{L}\mathrm{ax}_{\mathrm{N}}(\mathbb{V}\mathcal{B}, \mathbb{C}\mathrm{at}) \simeq \mathbb{L}\mathrm{ax}(\mathbb{V}\mathcal{B}, \mathbb{S}\mathrm{pan}) \simeq \mathbb{L}\mathrm{DF}/\!\!/\mathcal{B}$.

(Joint with J.R.B. Cockett and R.J. Wood)

Keith O'Neill (Ottawa): "Differential Forms for T-Algebras in a Kahler Category"

Abstract: A Kahler Category is an additive symmetric monoidal category with an algebra modality in which the free algebra associated to each object is assigned a universal derivation. This is done in analogy with the notion of Kahler differentials in algebraic geometry. In this talk I will be discussing the existence of modules of differential forms associated to associative algebras arising from the T-algebras of a Kahler category.

Simona Paoli (Leicester): "Categorical models of homotopy types"

Abstract: Homotopy n-types are CW complexes whose homotopy groups vanish in dimension higher than n. They can be thought of as the building blocks of topological spaces, thanks to a classical construction, the Postnikov decomposition of a space. The problem of finding algebraic models of homotopy types is a fundamental question in homotopy theory, which leads to notions in higher category theory.

In the first talk, I will cover the basic homotopical background concening n-types, and I will give a brief survey of their algebraic modelling. In the second talk, I will illustrate one such algebraic model, called n-typical n-fold groupoids. The latter is based on n-fold internal categorical structures, and has desirable properties. The second talk is joint work with David Blanc.

Bob Paré (Dalhousie): "Colimits and Profunctors"

Abstract: We study an application of the theory of profunctors to colimits. In particular we isolate a class of profunctors, which we call total, which give the most general (in a sense we make precise) morphism of diagrams which induces a morphism between the colimits. We investigate the consequences of this definition. Profunctors are the relations between categories. There is also a corresponding notion of "single valued" profunctor, which together with total is equivalent to representability.

Laura Scull (Fort Lewis): "Equivariant homotopy and diagram categories"

Abstract: In studying spaces with group actions, it has proved particularly fruitful to consider them not just as spaces but as diagrams of spaces, where the basic diagram is given by the various fixed point sets of the space under the subgroups of the acting group. This diagram approach of Bredon leads to algebraic invariants and cohomology theories which are defined using categorical constructions of functors and natural transformations. Studying the structures best suited to capturing the geometry leads to some interesting categorical constructions. I will discuss my current research, joint with Dr. D. Pronk, in using category theory to create equivariant cohomology theories for local coefficients.

Robert Seely (McGill): "Towards a notion of Cartesian differential storage category"

Abstract: Cartesian differential categories were introduced to provide a direct axiomatization of the differential structure of the coKleisli category of a (tensor) differential category. In particular, it was proved that the coKleisli category of any (tensor) differential (storage) category is always a Cartesian differential category. However, Cartesian differential categories arise in many different and independent ways so that the converse is clearly not true. However, it is reasonable to ask whether every Cartesian differential category admits a full and faithful embedding into the coKleisli category of some (tensor) differential category.

The purpose of this talk is to show our recent progress towards achieving such embedding theorems. This has involved developing a general theory of "storage" categories in order to have a suitably abstract characterization of the target of these representation theorems.

Joint work with R. Blute and J.R.B. Cockett

Takeo Uramoto (Kyoto): "On Tannaka dualities"

Abstract: Roughly speaking, Tannaka duality is a duality between coalgebras (we mean comonoids) and categories, bialgebras and monoidal categories, and Hopf algebras and monoidal categories with duals. The aim of this talk is to introduce a use of Tannaka duality theorem through solving a particular problem motivated in concurrency theory based on the theory of universal coalgebras. The emphasis of this talk is not put on the original motivation and the results themselves, but on Tannaka duality as our tool, and explaining the reason why we need it. Concretely, for instance, how can we solve the folllowing?: (1) How many monoidal structures can exist on a given category of interests? Is the number of them finite or infinite? (2) Can the category be autonomous with respect to some monoidal structure on it? In our study, we were concerned with these problems for the category of automata and simulations. But it seems difficult to deal with these category theoretic problems directly. Fortunately, by proving and using Tannaka duality (in Rel, not in Vect), these categorical problems were translated into those of combinatorics of finite words, and the answers are: (1*) the category of automata possesses at most 2ˆ(nˆ3 + n) monoidal structures if the set of alphabets has n elements. (2*) No, because a particular coalgebra consisting of finite words in Rel cannot be Hopf algebra in Rel w.r.t any bialgebra structure.

Masuka Yeasin (Calgary): "Linear functors and their fixed points"

Abstract: A linearly distributive category with a monoidal category acting on it both covariantly and contravariantly is called a linear actegories and provides a basic model for concurrent programming. The two actions give the structure of a parameterized linear functor and form a basis on which one can build initial (and final) concurrent data or protocols.

We shall prove that when data is built on a linear functor that the initial and final data types form a linear functor pair. In particular, we shall illustrate the value of circuit (or string) diagram proofs to establish these facts.