

Jigsaw Activity - Symmetry

Each of the following problems can be solved very elegantly by using symmetry. See if you can solve these problems without using any messy or complicated mathematics. In your group, just work on the three problems assigned to you:

Group A - Problems 1 through 3

Group B - Problems 4 through 6

Group C - Problems 7 through 9

Group D - Problems 10 through 12

1. A rectangular box has faces with areas 15, 18, and 30. Determine its volume. Also determine the dimensions of the box.
2. Chantel and Vernon play the following game on a large circular table. They take turns placing a penny anywhere on the table, with the rule that this penny cannot touch any other penny already on the table. Eventually one person will not be able to place a penny anywhere on the table, in which case s/he will lose. Suppose that Chantel moves first. Does either player have the winning strategy? If so, which player? Explain.

3. Determine the value of

$$\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$$

4. Determine the value of

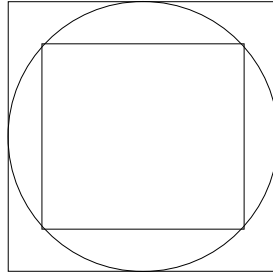
$$\int_{-1.23}^{1.23} \tan x \, dx$$

5. Determine the value of

$$\sum_{i=1}^{100} \frac{i^3}{(101-i)^3 + i^3}$$

6. Each day a woman's husband picks her up at the train station to drive her home. One day she arrives at the station an hour early and begins to walk home along the road that her husband always takes. He meets her en route and takes her the rest of the way home. Had she waited at the station, he would have picked her up on time, and gotten home at the usual time. However, on this day, she got home twenty minutes earlier than normal. How long did she walk? (You can assume that the speed of the car is constant).
7. Determine the value of $1 + 2 + 3 + \dots + 100$. (If you know a formula for the sum of the first n positive integers, can you prove why the formula is true?)
8. The Leafs and the Canadiens are playing in the Stanley Cup playoffs. Assume both teams are evenly matched, so each team has a fifty percent chance of winning each game (if you follow hockey, you know that this is one very bad assumption). The first team to win four games wins the series. Which is the more likely scenario, that the series goes to six games, or that the series goes to seven games?
9. Find the minimum perimeter of a triangle with one vertex at $(7, 1)$, one vertex on the x-axis, and one vertex on the line $y = x$.

10. A square is inscribed in a circle, and this circle is inscribed in a square. Determine the ratio of the areas of the two squares.



11. Alexander the Mayor is holding two glasses which are identical in size. He fills glass A with juice so that it is half full, and he fills glass B with Keith's so that it is also half full. He then takes a teaspoon of Keith's from glass B and pours it into glass A. Then he mixes up glass A a bit, and pours a teaspoon of that mixture into glass B. Now there is a little bit of Keith's in glass A, and a little bit of juice in glass B. Alexander the Mayor says that there is more Keith's in glass A than there is juice in glass B. Is he correct?
12. Two poles, with heights 4 and 12 metres, are 20 metres apart. A guy wire stretches from the top of each of the poles to some point P on the ground. Where should point P be located to minimize the total length of the wire?

