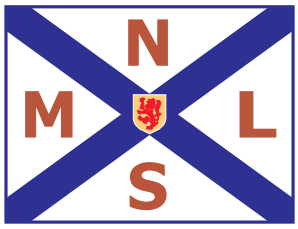



Nova Scotia
Math League

2011–2012

## Game Two

# PROBLEMS

## Team Questions

1. Alan leaves Halifax at 8am and drives at 60 km/h. Bob leaves Halifax at 9am and drives at 100 km/h along the same road as Alan.

   At what time does Bob catch up with Alan?

2. Define sets $S_1, S_2, S_3, \ldots$ as follows:

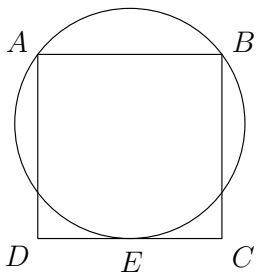
   $$S_1 = \{1\}, \quad S_2 = \{2,3\}, \quad S_3 = \{4,5,6\}, \quad S_4 = \{7,8,9,10\}, \quad \text{etc.}$$

   Find the sum of the elements of $S_{10}$.

3. How many digits does the number $5^{80}8^{30}$ have when it is expressed as usual in decimal form?

4. A race involves three horses. In how many ways can they finish, including ties?

5. In the figure below, $CD$ is tangent to the circle at $E$, and $ABCD$ is a square. If the circle has radius 1, what is the area of the square?



6. A rectangular box is to have sides of **integer** lengths (measured in centimetres) and a volume of 120 $\text{cm}^3$. What is the minimum surface area of the box, in square centimetres?

7. Point $P$ is on the line $y = 2x - 2$ and point $Q$ is on the line $y = 2x + 3$. What is the shortest possible distance between $P$ and $Q$?

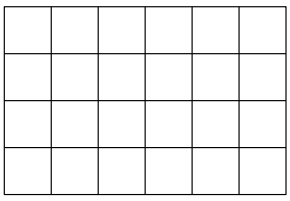
8. Bobby's sock drawer contains a total of ten socks, 5 red and 5 blue. He selects two socks at random and tells you that at least one of them is red. What is the probability that both of the chosen socks are red?

9. Let $f_0(x) = x$, and for $n \geq 1$ define $f_n(x)$ recursively by the formula

$$f_n(x) = \frac{1}{1 - f_{n-1}(x)}.$$

Evaluate $f_{2012}(2012)$.

10. How many rectangles can be found in the figure below? (Note that a square is a rectangle!)



## Pairs Relay

P-A. Let A be the number of **odd** integers between 1 and 100 (inclusive) that have distinct digits.

Pass on A

P-B. You will receive A.

Suppose $x + y + z = 10$ and $xy + yz + zx = \texttt{A}$.

Let $\texttt{B} = x^2 + y^2 + z^2$

Pass on B

P-C. You will receive B.

Let C be the area of the triangle bounded by the $y$-axis and the lines $y = \texttt{B}x + \texttt{B}$ and $y = 4x + 4$.

Pass on C

P-D. You will receive C.

An ice-cream cone costs C dollars. Let D be the number of ways one can pay for the cone using only loonies, quarters, and dimes.

Done!

## Individual Relay

I-A. Jack is 40% heavier than Sam. Sam is 25% lighter than Ed.

Therefore Jack is A% heavier than Ed. Pass on A

I-B. You will receive A.

Let B be the sum of the roots of the equation

$$(x-1)(x-2)+(x-2)(x-\mathtt{A})=0.$$

Pass on B

I-C. You will receive B.

Solution X is created by dissolving 7.5 grams of salt in B litres of water. Solution Y is created by dissolving 11 grams of salt in 4 litres of water. Solution Z is created by mixing 3 litres of Solution X with 2 litres of Solution Y.

Let C be the number of grams of salt in 1 litre of Solution Z. Pass on C

I-D. You will receive C.

Let D be the **denominator** when the fraction

$$\frac{2^{\mathtt{C}}-2^{-\mathtt{C}}}{4^{\mathtt{C}}-4^{-\mathtt{C}}}$$

is expressed in lowest terms. Done!