Nova Scotia
Math League

2013–2014

Game One

**PROBLEMS**

Team Questions

1. Evaluate:

$$2(2(2(2(2(2(2(2(2(2(2+1)+1)+1)+1)+1)+1)+1)+1)+1)+1)+1$$

2. Two trains of lengths 200m and 220m run on parallel tracks. When the trains are travelling in the same direction, one completely passes the other in 30 seconds. When travelling in opposite directions, the trains completely pass each other in 7 seconds.

   Find the speed of the **slower** train, measured in metres per second.

3. The figure below was created by drawing four circles of radius 1, one centred at each of the vertices of a unit square. Find the perimeter of this figure (that is, the length of solid curve in the diagram).
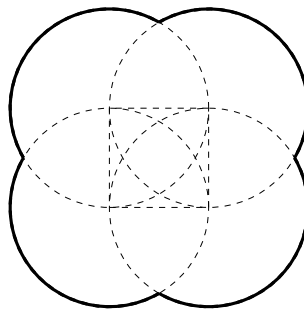
4. Real numbers x, y, z and w satisfy the system of equations

$$x + y + z = 1$$
$$y + z + w = 2$$
$$z + w + x = 3$$
$$w + x + y = 4.$$

   Find the value of x.

5. When five circles of the same diameter are drawn in the plane, what is the maximum number of points of intersection?

6. In square $ABCD$, a line is drawn from vertex A to the midpoint of side BC, and another is drawn from vertex C to the midpoint of side AB. These lines intersect at a point P inside the square. If the square has area 36 square units, what is the area of quadrilateral $APCD$?
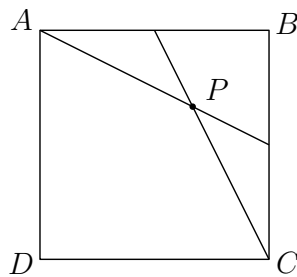

7. A bowl contains punch that is 40% juice; that is, every 100mL of punch contains exactly 40mL of juice. If one ladle of punch were removed from the bowl and replaced with one ladle of juice, then the modified punch would be $p\%$ juice. But this same concentration of juice could also be obtained by adding 3 ladles of juice to the original bowl of punch.

Determine p.

8. Andy and Bob take turns rolling a six-sided die, with Andy making the first roll.

   What is the probability that Andy will be the first person to roll a 6?

9. A circle passes through the vertex and both x-intercepts of the parabola $y = x^2 - 14x + 40$. A line passes through the origin and is tangent to the circle at a point P. Find the distance from P to the origin.
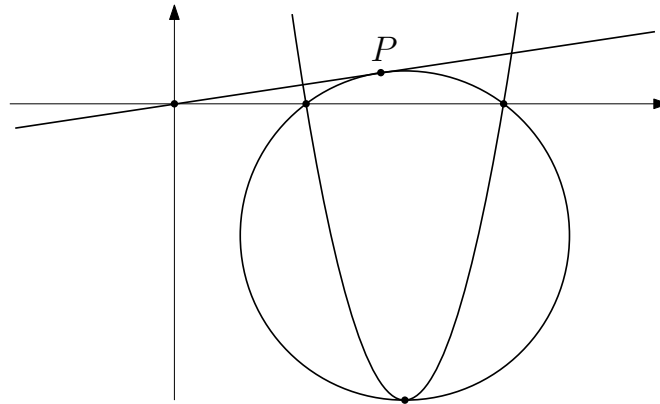
10. Find the number of triangles in the figure below.
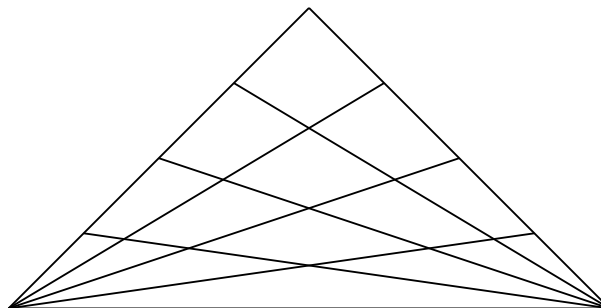
Pairs Relay

P-A. A town has 1000 residents, each of whom voted on two motions in a referendum. After tallying the votes, it was found that 744 residents supported the first motion, while 526 supported the second. Only 32 residents voted against both motions.

Let `A` be the number of residents who voted in favour of both motions.

Pass on `A`

P-B. You will receive `A`.

Let `B` be the `A`-th digit to the right of the decimal point in the decimal expansion of $\frac{1}{7}$.

Pass on `B`

P-C. You will receive `B`.

Let `C` be the unique digit such that `1C2C3C4C5B` is divisible by 9.

Pass on `C`

P-D. You will receive `C`.

Let `D` be the unique value of x that satisfies the following equation:

$$\frac{x + \text{C}}{x + \text{C} + 1} - \frac{x + \text{C} - 1}{x + \text{C}} = \frac{1}{(x - \text{C})^2}.$$

Done!

Individual Relay

I-A. Let A be the units digit of $2013^{2014} - 2014^{2013}$.

I-B. You will receive A.

Suppose $a_0 = $ A and $a_1 = $ A, and let $a_n = 4a_{n-1} - 3a_{n-2}$ for $n \geq 2$.

Let B $= a_{2013}$.

I-C. You will receive B.
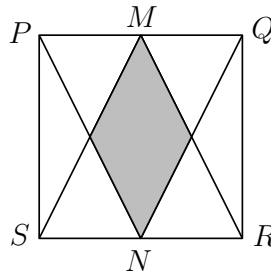
Let α and β be the two roots of the equation

$$x^2 - Bx + B = 1.$$

Let C $= \alpha^2\beta + \beta^2\alpha$.

I-D. You will receive C.

In the figure below, square $PQRS$ has perimeter C, and M and N are the midpoints of PQ and RS, respectively.

Let D be the area of the shaded region.