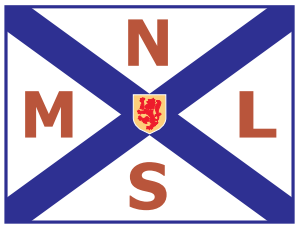


Nova Scotia
Math League

2013–2014

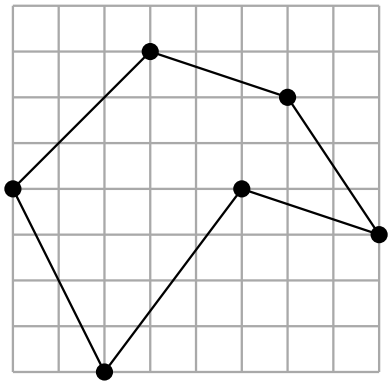
Game Two

PROBLEMS

Team Questions

1. Find the final digit of 9^{9^9}.

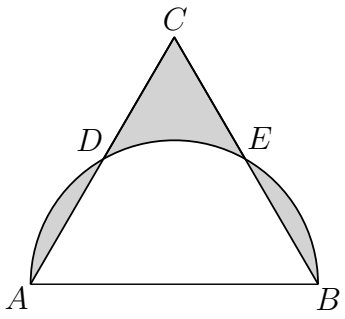
2. Six points in a square grid are joined to form a hexagon, exactly as shown below. Assuming each square of the grid has area 1, find the area of the hexagon.



3. Each side of a cube is painted either red or blue. How many different painted cubes are possible?

   Note: Two painted cubes are the same if one can be rotated to get the other. For instance, there is only one cube whose faces are all red.

4. In the figure below, $\triangle ABC$ is equilateral with sides of length 2, and a semicircle with diameter AB meets the triangle at D and E. Find the area of the shaded region.



5. Alice and Mary each pick an integer at between 1 and 10, inclusive. If the girls make their choices at random, what is the probability that their numbers will differ by at most 2?

6. Megan decides to pass some time by writing down all possible 5-letter "words" that can be formed by rearranging the letters of her name. She lists the possibilities in alphabetical order. At what position in her list does her name appear?

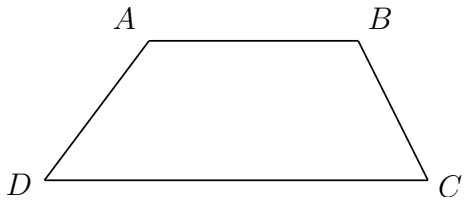
7. Find the slope of the line passing through the points where the circles

$$x^2 + 2x + y^2 - y = 8 \qquad \text{and} \qquad x^2 - x + y^2 - 2y = 9$$

intersect.

8. The line $y = 2x + 6$ intersects the parabola $y = 10 - x^2$ at two points, A and B. Find the length of line segment AB.

9. Trapezoid $ABCD$ has parallel sides AB and CD of lengths 6 and 10, respectively, with base angles $\angle ADC = 40^\circ$ and $\angle BCD = 50^\circ$. Find the length of the line segment joining the midpoints of AB and CD.



10. A school with a total of 442 students held an assembly, but not everybody showed up. Two thirds of the students who skipped the assembly were boys. Of the students who attended the assembly, four sevenths were girls, two fifths were seniors, and one quarter fell asleep.

    How many students were at the assembly?

Pairs Relay

P-A. A store holds a "We Pay The Tax" event, during which they discount `A`% off their regular price but then charge `A`% tax on the discounted price. During this event, a customer can purchase a television priced at \$500 for a total of only \$495.

Pass on `A`

P-B. You will receive `A`.

A man runs twice around the same loop and covers a total distance of `A` kilometres. His average speed over the first loop is 10 km/h, and his average speed over the entire run is 12 km/h.

Let `B` be his average speed (in km/h) over the second loop.

Pass on `B`

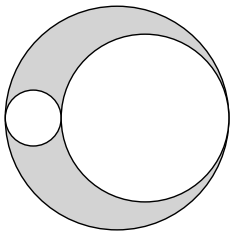
P-C. You will receive `B`.

When the number `C` is removed from the list $1, 2, 3, \ldots,$ `B`, the average of the remaining numbers is `C`.

Pass on `C`

P-D. You will receive `C`.

Three circles are arranged as in the diagram below, with the small and medium circles being tangent to each other and also to the large circle. The diameter of the large circle is `C`, and the area of the medium circle is 9 times the area of the small circle.



Let `D` be the area of the shaded region.

Done!

Individual Relay

I-A. Bob buys widgets at a cost of 3 for \$5, and sells them at 2 for \$5. Let A be the number of widgets Bob must sell to make a profit of \$50.

Pass on A

I-B. You will receive A.

Today is Jack's birthday. The sum of Jack's age (in years) with the ages of his two sons is A. But on Jack's A-th birthday, the sum of his and his sons' ages will be 2A.

Let B be Jack's age today.

Pass on B

I-C. You will receive B.

Fifteen men can dig a 100 metre trench in 6 hours. Let C be the number of hours it would take twelve men to dig a B metre trench.

Pass on C

I-D. You will receive C.

The operation $\star$ is defined by the formula

$$x \star y = xy + x + y.$$

For example, $2 \star 5 = 2 \cdot 5 + 2 + 5 = 17$.

Let $\mathtt{D} = \mathtt{C} \star (\mathtt{C} \star (\mathtt{C} \star \mathtt{C}))$

Done!