Nova Scotia Math League

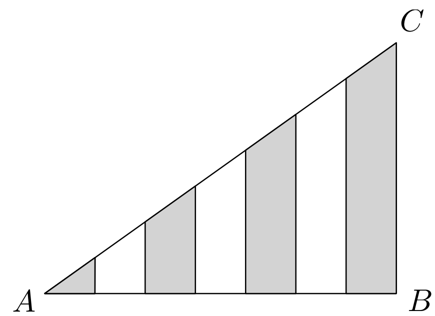
2014–2015

Game One

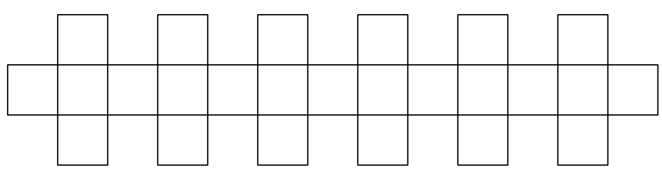
PROBLEMS

Team Questions

1. Right triangle ABC is divided into seven trapezoids of equal thickness, as shown below. Given that $|AB| = 4$ and $|BC| = 3$, determine the area of the shaded region.

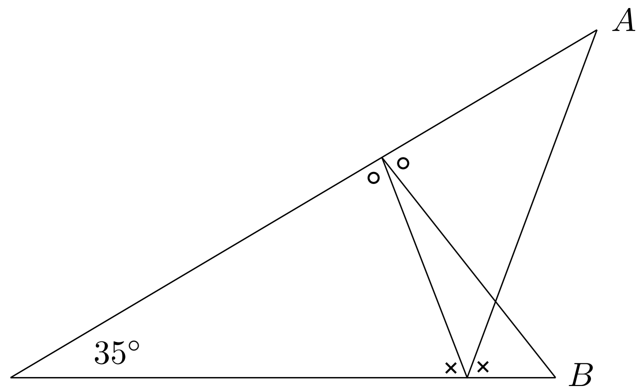


2. The shape below was created by pasting together 25 unit squares. When a similar shape is created with n squares, its perimeter is 100 units. Determine n.



3. A broken photocopier can only make copies that are either 75%, 100%, or 160% the size of the original. However, various other output sizes can be achieved by making copies of copies. What is the minimum number of copies required to produce one that is 108% the original size?

4. In the figure below, the two angles marked with circles are equal, as are those marked with crosses. Determine the **sum** of the (acute) angles at A and B.



5. A old table of factorials contains the line

$$20! = 243290X008176640000,$$

where X represents a digit that cannot be read because the ink has smudged. Determine X.

Note: Recall that the *factorial* of a positive integer n is the product $n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$. For instance, $4! = 4\cdot 3\cdot 2\cdot 1 = 24$.

6. Find all real numbers x such that

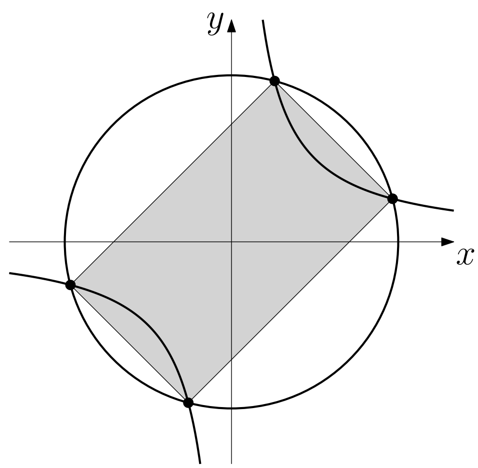
$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{x}}} = 1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{x}}}}}$$

7. Bob and Suzy each toss a fair coin 3 times in a row. What is the probability that Bob tosses fewer heads than Suzy?

8. Suppose $a = 1 + a^2$. Determine a^{60}.

9. How many different 4-digit numbers can be formed using only the digits 1, 2, and 3, if consecutive digits must differ by at most 1.

10. The circle $x^2 + y^2 = 36$ and the hyperbola $xy = 9$ intersect at exactly four points, as shown below. Find the area of the shaded region.



Pairs Relay

P-A. For certain numbers m and n we have the identity

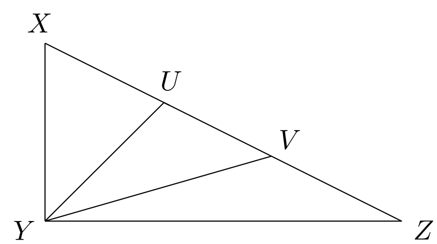
$$\frac{7x+19}{x^2+5x+6} = \frac{m}{x+2} + \frac{n}{x+3}.$$

Let $\mathtt{A} = mn$.

$\boxed{\text{Pass on } \mathtt{A}}$

P-B. You will receive $\mathtt{A}$.

Right triangle XYZ has legs $|XY| = 3$ and $|YZ| = \mathtt{A}$. The hypotenuse XZ is subdivided into 3 equal parts by points U and V.



Let $\mathtt{B}$ be the area of triangle UYV.

$\boxed{\text{Pass on } \mathtt{B}}$

P-C. You will receive $\mathtt{B}$.

Suppose X is 10% of Y, Y is 25% of Z, and W is $\mathtt{B}$% of Z.

Let $\mathtt{C} = \dfrac{W}{X}$.

$\boxed{\text{Pass on } \mathtt{C}}$

P-D. You will receive $\mathtt{C}$.

Stations 1 through 9 are marked in clockwise order around a circle. Movement between stations is governed by the following rule: From station n you must move 2 stations clockwise if n is divisible by 3, and 1 station clockwise if not.

Begin at station $\mathtt{C}$ and let $\mathtt{D}$ be the final station after making 100 moves.

$\boxed{\text{Done!}}$

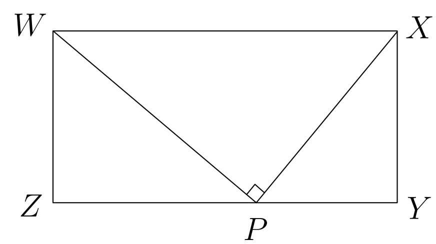
Individual Relay

I-A. Let A be the number of ways the letters of APPLE be rearranged so the first and last letters are consonants.

$\boxed{\text{Pass on } \mathtt{A}}$

I-B. You will receive $\mathtt{A}$.

Right triangle WPX is inscribed in rectangle $WXYZ$, which has sides $|WX| = \mathtt{A}$ and $|XY| = 8$.



Let $\mathtt{B}$ be the area of $\triangle WPX$.

$\boxed{\text{Pass on } \mathtt{B}}$

I-C. You will receive $\mathtt{B}$.

Let $\mathtt{C}$ be the number of positive integers n that satisfy the inequality $n^2 + 1000 < \mathtt{B}n$.

$\boxed{\text{Pass on } \mathtt{C}}$

I-D. You will receive $\mathtt{C}$.

The degree measure of each interior angle of a regular polygon is an integer multiple of $\mathtt{C}$.

Let $\mathtt{D}$ be the number of sides of this polygon.

$\boxed{\text{Done!}}$