Nova Scotia
Math League
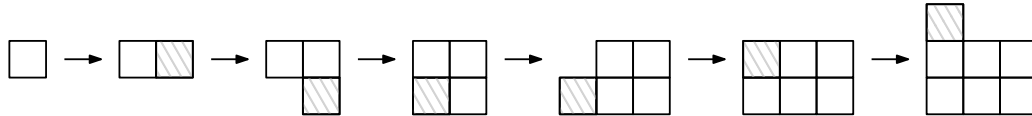
2018–2019

Game Three

PROBLEMS

Team Questions

1. A sequence of figures is obtained by successively adjoining 1 cm × 1 cm squares in a spiral pattern, as shown below. (The striped square is the most recently added.)


   What is the perimeter of the figure whose area is 750 cm^2?

2. Three numbers sum to 2019. If one of the numbers is doubled, the sum triples. And if one of the numbers is quadrupled, the sum doubles.

   Find the numbers.

3. Each small rectangle in the figure below is similar to the large rectangle. If the perimeter of the large rectangle is 16, what is the perimeter of each small rectangle?
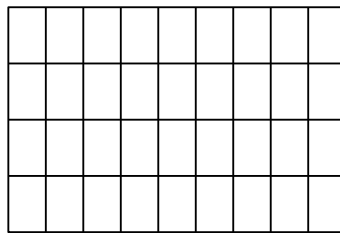
4. How many integers between 1 and 100 (inclusive) **cannot** be written as a sum of 10 consecutive integers?

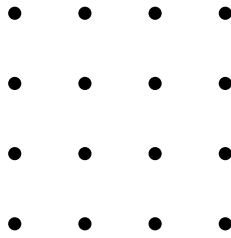   *Note:* The summands can be negative.

5. Three circular buttons are placed tightly in a rectangular box as shown below. The radii of the buttons are 1 cm, 2 cm, and 3 cm and the box has a width of 8 cm, as indicated. Find the length of the box.
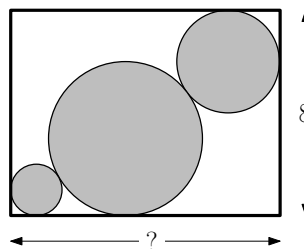
6. The empty cells of the square below are to be filled with positive real numbers such that the product of the three entries along each row, column, and diagonal is 1. Find x.

	9	
		4
		x

7. How many rectangles can be formed by joining four dots in the 4×4 square lattice?

$$\begin{matrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{matrix}$$

8. The sequence $a_0, a_1, a_2, \ldots$ satisfies $a_n = a_{n-1} + a_{n+1}$ for $n \geq 1$.

If $a_1 = 2019$ and $a_{2019} = 1$, find a_{2000}.

Pairs Relay

P-A. My brother is 5 years older than me, and I am 2 years older than my sister. In seven years our average age will be twice what it is now.

Let A be my current age. [Pass on A]

P-B. You will receive A.

Let $n!$ denote the product of all positive integers less than or equal to n. For example, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$.
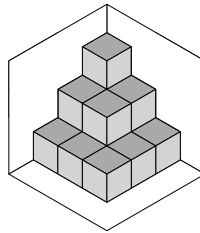
Calculate:
$$B = \frac{(A+1)! - A!}{(A-1)!}$$

[Pass on B]

P-C. You will receive B.

Unit cubes are stacked in the corner of a room to form a pyramid, as shown below.

The stacking continues in this manner until the bottom layer contains B cubes. Let C be the exposed surface area of the resulting pyramid (*i.e.* the area that can be seen from inside the room).

[Pass on C]

P-D. You will receive C

A rectangle with perimeter C has been tiled with congruent rectangles in the manner illustrated below.
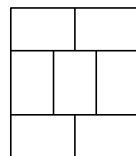
Let D be the perimeter of each small rectangle. [Done!]

Individual Relay

I-A. Let A be the number of points with integer coordinates that lie on the line segment joining $P = (-8, -13)$ and $Q = (10, 11)$.

Note: Include P and Q in your count! $\boxed{\text{Pass on A}}$

I-B. You will receive A.

A collection of identical unit cubes are fastened together to form a large $A \times A \times A$ cube. The big cube is painted on all sides and then separated again into the original cubes. Let B be the number of these small cubes which remain completely unpainted.

$\boxed{\text{Pass on B}}$

I-C. You will receive B.

Let C be the number of ways the digits $2, 8$ and $\sqrt[3]{B}$ can be rearranged to form a three-digit number divisible by 3.

$\boxed{\text{Pass on C}}$

I-D. You will receive C.

Two sides of an isosceles triangle have lengths C and C/3.

Let D be the area of the triangle. $\boxed{\text{Done!}}$

Team Questions Answer Key

1. 110

2. $\{4038, 673, -2692\}$.

3. $\dfrac{8}{3}$

4. 90

5. $7 + 2\sqrt{3}$

6. $\dfrac{3}{2}$

7. 44

8. 2020

Pairs Relay Answer Key

A. 6

B. 36

C. 78

D. 30

Individual Relay Answer Key

A. 7

B. 125

C. 6

D. $\sqrt{35}$