Algebra

originated as the solution to problems involving an unknown number. This sort of problem was studied as early as 1800-1600 B.C. Polynomial equations were the natural types of equation to consider. These led to the development of complex numbers — certain equations such as $x^2 + 1 = 0$ have no solutions using usual numbers. However, from the equation, we can say a lot about arithmetic properties of an imagined solution, in much the same way that we can understand negative numbers using an equation such as $x + 3 = 0$. In time, these new numbers were accepted as a standard part of mathematics, and proved to be invaluable for many mathematical problems. The solution to $x^2 + 1 = 0$ was named i. [There are two solutions to this equation, and either one can be chosen to be named i, the other is $-i$.] Then using addition and multiplication, we can obtain numbers of the form $a + bi$, where a and b are real numbers. Such numbers are called complex numbers. Addition of complex numbers is obvious, while multiplication uses the equation $i^2 + 1 = 0$.

The question naturally arises whether we need to introduce more numbers to solve more advanced forms of equations. The *fundamental theorem of algebra* tells us that this is not the case — once we have the complex numbers, all polynomials have solutions.

Starting in the 19th century, mathematicians began to consider the use of algebra where the unknowns could be things other than numbers. The first example of this is the algebra of matrices, ...

Later examples include groups, rings, fields. The study of general fields is useful in studying solutions to polynomials. For a polynomial, one can consider a field which contains the coefficients of the polynomial. One can then consider a larger field which also contains the roots of the polynomial, and consider how the smaller field fits within the larger field. This technique (known as Galois theory) was used to show that there is no formula for solving a general quintic equation. It also showed the impossibility of three geometrical constructions attempted by the ancient Greeks.

Another area of study within algebra is universal algebra, which looks at general methods for studying algebraic structures. A particularly important algebraic structure for this is a lattice.