Analysis

Deals with notions related to rates of change of functions. Most basic notion is a continuous function. Roughly speaking, a function is continuous if you can use the value of the function at a nearby point as a good approximation for the value of the function at the point of interest. The detailed definition of continuity (and even function) were developed in the 19th century, but mathematicians had an intuitive sense of these notions long before then.

An example of the sort of result that can be found for continuous functions is the *intermediate value theorem*, which states that a continuous function which takes the values a and b must take every value between them.

One of the main subjects of analysis is calculus. This studies the notions of rates of change, and summing up the rate of change of a quantity to get the total quantity - which happens to correspond to finding the area of a shape. The idea for calculating the rate of change of a function is to consider how much it changes in a small time, and see what happens as this small time gets smaller.

Rates of change (called derivatives) are very important for a number of reasons. They can be natural quantities in their own right, for example the speed of an object is the rate of change of its position. Often scientific laws take the form of equations involving some quantity and its derivative. These equations are called *differential equations*, and studying the properties of these equations is useful for a number of applications of mathematics.

Rates of change can also be important tools for studying the original function. For example, if the function has a maximum (or minimum) value, then at that value, the rate of change must be zero. Therefore, in order to find the maximum (or minimum) value of a function, one can restrict attention to the cases where the derivative is zero.

The study of calculus can be applied equally well to complex numbers, and it turns out that the theory is particularly nice. For the derivative of a function of complex numbers to exist is a much stronger restriction than the corresponding condition for real numbers. This means that there are far more results about complex differentiable functions than about real differentiable functions. This means that the study of complex analysis is more than idle curiosity — it actually provides profound tools for studying functions unrelated to complex numbers.

Another branch of analysis is *harmonic analysis*, which attempts to study methods of approximating functions using basic waves. These basic waves are important as they correspond to fixed pitches of sound. They are also solutions to particular simple differential equations (actually, this is the reason that they occur as the sound of these fixed pitches). It turns out that any periodic function can be expressed as a sum of these basic periodic functions (sine waves). This sum is called the *Fourier series* of the function, and is important for a number of reasons. Musically, it expresses a sound as a combination of a number of multiples of the base pitch. A more general form for functions that are not periodic is the *Fourier transform*, which can help to solve certain differential equations, because the complicated operation of taking the derivative corresponds to a much simpler operation on the Fourier transforms, thereby changing the problem of solving a differential equation into solving an algebraic equation.