

Geometry

Deals with shape. Largely developed in ancient Greece, with Euclid's Elements covering large quantities of what was known to them.

Major upheaval following Descartes' invention of coordinates. These allowed geometrical shapes to be described by algebra, and problems in geometry could all be solved algebraically. This algebraic model of geometry was also important for the development of algebra and analysis, because it gave an intuitive geometrical model of those subjects. Abstract notions such as integration (finding a function given its rate of change) could be described in terms of finding areas of regions.

This algebraic model of geometry paved the way for mathematicians to better understand the fundamentals of the subject. For a long time Euclid's *parallel postulate* that "given a line and a point not on the line, there is exactly one line through the point parallel to the given line" was considered less obvious than the other axioms, and mathematicians wondered whether it could be shown to be a consequence of the other axioms. By producing a different type of algebraic model, mathematicians were able to create geometries which satisfied the other axioms but not the parallel postulate. These geometries are important for complex analysis, and also in relativity, where physicists believe that these geometries are actually a more accurate description of our universe than Euclid's geometry.

The algebraic model also allowed new branches of geometry to develop. Now that algebraic equations have a geometric interpretation, we can study the geometry of these shapes. The modern study of this type of algebraic geometry is rather abstract, but it originates from the geometry of these curves. For studying these curves, projective geometry is a useful tool. In projective geometry, we extend Euclidean geometry by adding a single "line at infinity". The benefit of this is that now any two lines intersect in a point, so there are no special pairs of lines. In fact, the number of points of intersection of any two curves is determined by their degrees.

Another way in which we can extend Euclidean geometry is in terms of dimensions. The real world is 3-dimensional, but mathematically it is possible to study geometry in any number of dimensions.

Another branch of geometry is the differential geometry, which deals with shapes up to smooth transformations. Other more abstract forms of geometry are *metric spaces*, which give an abstract model of distances, and *finite geometries*, which give an abstract model of points and lines.