MATH 1001, Mathematics for Liberal Arts Students
WINTER 2012
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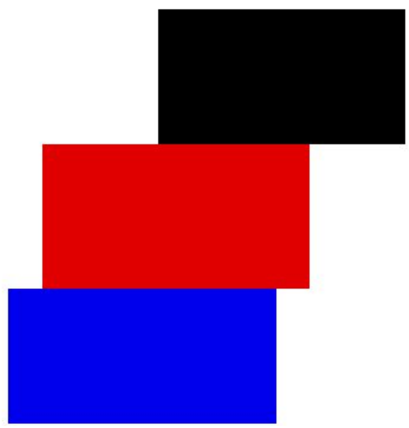
Fun Problems 1

These problems are to give you a chance to experience the thrills and frustrations of problem-solving for yourselves. They are not for credit, but I hope you will find the experience worthwhile.

If the wording of any problem is ambiguous, feel free to interpret it in whatever way makes the problem better (more interesting, easier to approach, ...)

1. A child is playing with 3 wooden blocks. Each block is 5cm long. The child stacks the blocks in a pile like so:



   How far can the top block extend beyond the end of the bottom block?

2. Is it possible to fit one rectangle inside another rectangle in such a way that the inner rectangle has a larger perimeter (sum of all side lengths)?

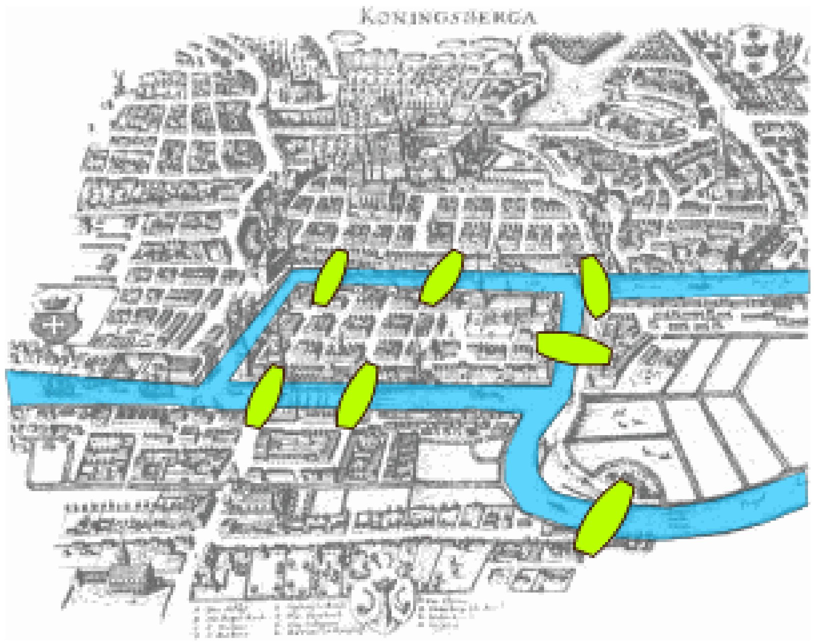
3. A (3×3) *magic square* is a 3×3 grid containing each of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 exactly once, such that the numbers in every row, column and diagonal sum to 15. For example, the following

8	1	6
3	5	7
4	9	2

   How many 3×3 magic squares are possible?

   [You will probably need a hint, but it doesn't do any harm to think about it before asking for the hint.]

4. A certain city has 7 bridges laid out as follows:

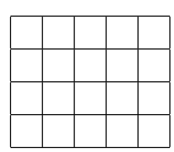


Is it possible to find a path that crosses every bridge exactly once, and starts and finishes on the same bank or island? [The only way to cross any part of the river is using one of the seven bridges — no swimming is allowed.]

5. (a) There are 6 people at a party. Show that either some three of these people all know one another, or some three of these people are all strangers. [Assume that if person x knows person y, then person y also knows person x.]

   (b) How many people are needed to ensure that there are either some four people who all know each other, or some three people who are all strangers.

6. How many rectangles are in the following picture? [Squares count as a special kind of rectangle.]



7. We have an 8×8 chessboard, and remove two opposite corners. Is it possible to cover the remaining squares with dominoes that exactly cover two adjacent squares, such that no two dominoes overlap, and no domino covers any area outside the board?

8. For two numbers x and y, let $x \otimes y$ denote the value $x + y + xy$. Start with a list of the numbers $1, 2, \ldots, 100$, and at each stage pick two of the numbers a and b in the list, and replace them by $a \otimes b$ (so that the new list has one fewer number). Repeat this process until there is only one number remaining. How many different answers can we get by choosing the numbers in different orders?

9. How does the length of a day vary with the angle between the earth's axis and the line between the earth and the sun? [The earth's axis is the line about which the earth rotates.]

10. The number 169 has a special property. It is a square number (13^2), it is the sum of 2 squares ($12^2 + 5^2$), it is the sum of 3 squares ($12^2 + 4^2 + 3^2$), and so on. How far does this pattern continue? That is, for which numbers n is it possible to write 169 as a sum of n squares of positive integers?

11. What is the last digit of the number $7^{1234567890}$? [This number is far too large for a calculator or computer. The point is that you can work out the last digit without working out the whole number.]