MATH 1115, Mathematics for Commerce
WINTER 2011
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Sample Final Examination

Calculators are not permitted.

Each multiple choice question is worth one mark, other questions are worth two marks. Include all your working for the other questions, but for multiple-choice questions, it is sufficient to just indicate the letter. [Make it clear which letter you mean as your answer. Ambiguous answers will be marked wrong.]

1. It is impossible to pay off a loan for $20,000 at interest rate 6% compounded monthly, if the monthly payments are less than:

   (A) 20000×1.005

   (B) 20000×0.005

   (C) 20000×0.06

   (D) $\frac{20000}{1.005}$

   (E) Any monthly payments will eventually pay off the loan.

2. A company is considering investing $100,000 in a new project. They estimate that the project will lead to the following net cash flows at the end of each year:

End of Year	Cash flow
1	$20,000
2	$50,000
3	$30,000
4	$30,000

   The net present value of the project at 10% interest compounded annually is given by:

   (A) $20000 + 50000 \times 1.1^{-1} + 30000 \times 1.1^{-2} + 30000 \times 1.1^{-3}$

   (B) $20000 \times 1.1 + 50000 \times 1.1^{2} + 30000 \times 1.1^{3} + 30000 \times 1.1^{4} - 100000$

   (C) $20000 \times 1.1^{-1} + 50000 \times 1.1^{-2} + 30000 \times 1.1^{-3} + 30000 \times 1.1^{-4}$

   (D) $20000 \times 1.1^{-1} + 50000 \times 1.1^{-2} + 30000 \times 1.1^{-3} + 30000 \times 1.1^{-4} - 100000$

   (E) It is not possible to calculate the net present value from the information given.

3. The matrix $A = \begin{pmatrix} 1 & 2 & -3 & 3 & -2 & 4 & 0 \\ 1 & 3 & 1 & 3 & -1 & 8 & -2 \\ -1 & -3 & 0 & -1 & -3 & -6 & 5 \\ 2 & 6 & 2 & 5 & -2 & 17 & -2 \\ 2 & 5 & -2 & 6 & -4 & 14 & 0 \\ -1 & -3 & -1 & -3 & 1 & -7 & 1 \\ 1 & 4 & 5 & 2 & 0 & 13 & -1 \end{pmatrix}$ is invertible with inverse $A^{-1} = \begin{pmatrix} 113 & 60 & 11 & -37 & -48 & 53 & 62 \\ -44 & -26 & -4 & 18 & 17 & -22 & -26 \\ 9 & 5 & 1 & -3 & -4 & 4 & 5 \\ 3 & 3 & 0 & -4 & 0 & 1 & 3 \\ 5 & 3 & 0 & -4 & -1 & 2 & 4 \\ 1 & 1 & 0 & -1 & 0 & 1 & 1 \\ 1 & 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix}$.

Which of the following is part of the solution to the system of equations

$$\begin{array}{rrrrrrrcr}
a & +2b & -3c & +3d & -2e & +4f & & = & 2 \\
a & +3b & +c & +3d & -e & +8f & -2g & = & 3 \\
-a & -3b & & -d & -3e & -6f & +5g & = & 3 \\
2a & -4b & +2c & +5d & -2e & +17f & -2g & = & 2 \\
2a & +6b & -2c & +6d & -4e & +14f & & = & -1 \\
-a & -3b & -c & -3d & +e & -7f & +g & = & -1 \\
a & +4b & +5c & +2d & & +13f & -g & = & -2
\end{array}$$

(A) $a = 54$

(B) $b = -35$

(C) $c = 10$

(D) $e = 0$

(E) $f = 0$

4. The feasible region for the system of inequalities

$$\begin{aligned} 2x + 2y &\leqslant 8 \\ x - y &\geqslant 2 \\ x + 2y &\geqslant 1 \\ -2x + 4y &\geqslant 3 \end{aligned}$$

is:

(A) Bounded and $x = 3, y = 1$ is a corner.

(B) Empty

(C) Unbounded and $x = 7, y = -3$ is a corner.

(D) Bounded and $x = 7, y = -3$ is a corner.

(E) Unbounded and $x = 3, y = 1$ is a corner.

5. While applying the simplex method to solve a linear programming problem to maximise Z, you obtain the following simplex tableau.

x_1	x_2	x_3	s_1	s_2	s_3	s_4	Z	
1	0	-2	1	0	4	0	0	5
0	1	1	2	0	1	0	0	3
0	0	-2	-2	1	-1	0	0	8
0	0	1	3	0	2	1	0	11
0	0	3	2	0	1	0	1	15

What is the maximum value for Z?

(A) 10

(B) 15

(C) 20

(D) 23

(E) It is impossible to determine the maximum value for Z from the data given.

6. A company manufactures a certain product. The products produced have a $\frac{1}{1000}$ chance of being defective. If the company produces 3 products to fill one order, what is the probability that none of the three products is defective? [Assume that seperate products are independant.]

(A) $\frac{997}{1000}$

(B) $\frac{3}{1000}$

(C) $\left(\frac{999}{1000}\right)^3$

(D) $1 - \left(\frac{1}{1000}\right)^3$

(E) $1 - \left(\frac{999}{1000}\right)^3$

7. A second company needs some of the products produced by the company in the previous question. The probability that they need one is $\frac{1}{5}$. The probability that they need two is $\frac{1}{2}$ and the probability that they need three is $\frac{3}{10}$. What is the probability that none of the products they use is defective?

(A) $\left(\frac{999}{1000}\right)^2$

(B) $\frac{1}{5} \times \frac{999}{1000} + \frac{1}{2}\left(\frac{999}{1000}\right)^2 + \frac{3}{10}\left(\frac{999}{1000}\right)^3$

(C) $\frac{9979}{10000}$

(D) $\frac{1}{5}\left(\frac{999}{1000}\right)^3 + \frac{1}{2}\left(\frac{999}{1000}\right)^2 + \frac{3}{10} \times \frac{999}{1000}$

(E) 1

8. A company has invested in a new security system, which the suppliers assure them will only give a false alarm one night in every 5,000. From looking at police statistics, the company estimates that 1 building in every 1,001 is broken into every night. From these probabilities, what is the probability that there is a break-in in the building given that the alarm goes off? [Assume the alarm always goes off if there is a break-in.]

   (A) $\frac{1}{6}$

   (B) $\frac{2}{3}$

   (C) $\frac{4}{5}$

   (D) $\frac{5}{6}$

   (E) $\frac{300}{1001}$

9. two fair (6-sided) dice are rolled. The probability that the higher roll is 4 (or both rolls are 4) is:

   (A) $\frac{5}{36}$

   (B) $\frac{1}{6}$

   (C) $\frac{7}{36}$

   (D) $\frac{8}{36}$

   (E) $\frac{4}{11}$

10. Which of the following statments about limits are true:

    (i) If $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$ both exist, then so does $\lim_{x\to a} f(x)$.

    (ii) If $\lim_{x\to a} f(x)$ exists, then so do $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$.

    (iii) If $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist, then so does $\lim_{x\to a} f(x)+g(x)$.

    (iv) If $\lim_{x\to a} f(x)+g(x)$ exists, then so do $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$.

    (A) Just (iv)

    (B) (i) and (iii)

    (C) (ii) and (iv)

    (D) (ii) and (iii)

    (E) (i), (ii) and (iii)

11. Calculate $\lim_{x\to 1} \frac{x-1}{(x+2)(x-1)}$

    (A) 0

    (B) $\frac{1}{3}$

    (C) $\frac{1}{2}$

    (D) 3

    (E) ∞

12. Calculate $\lim_{x\to\infty} \frac{x+3}{x^2+7}$

(A) 0

(B) $\frac{3}{7}$

(C) 1

(D) ∞

(E) The limit does not exist and is not ∞

13. The function $f(x) = \begin{cases} \frac{x+5}{(x-2)(x-4)} & \text{if } x \leqslant 3 \\ \frac{1-x^2}{x-2} & \text{if } x > 3 \end{cases}$ is continuous:

(A) everywhere.

(B) everywhere except at $x = 3$.

(C) everywhere except at $x = -5$ and $x = 2$.

(D) everywhere except at $x = 2$, $x = 3$, $x = 4$.

(E) everywhere except at $x = 2$

14. The derivative of $\sqrt[3]{x^2 + 7x - 3}$ at $x = 3$ is:

(A) $-\frac{13}{27}$

(B) $\frac{13}{27}$

(C) $\frac{13}{9}$

(D) 39

(E) undefined

15. A manufacturer is selling a product. The demand equation is given by $p = 1000 - 5q - q^2$. The marginal revenue is given by:

(A) $-5 - 2q$

(B) $1000 - 5q - q^2$

(C) $1000 - 10q - 3q^2$

(D) $\frac{-1000}{q^2} - 1$

(E) 1000

16. For the demand function in the previous question, the point elasticity of demand at $p = 500, q = 20$ is:

(A) $\frac{5}{9}$

(B) $-\frac{5}{9}$

(C) -25

(D) $-\frac{1}{25}$

(E) -1

17. The function $f(x)$ satisfies $f'(5) = 0$ and $f''(5) = 2$. The point $x = 5$ is

(A) A local maximum for $f(x)$.

(B) A local minimum for $f(x)$.

(C) Definitely a global maximum for $f(x)$.

(D) Definitely a global minimum for $f(x)$.

(E) Neither a local maximum nor a local minimum.

18. The critical value of the function $f(x) = \frac{x^2-3}{x+2}$ at $x = -1$ is:

(A) a local (relative) maximum, but not a global (absolute) maximum.

(B) a global (absolute) maximum.

(C) a local (relative) minimum, but not a global (absolute) minimum.

(D) a global (absolute) minimum.

(E) Neither a local minimum nor a local maximum.

19. Which of the following lines is an asymptote to the function $f(x) = \frac{x^4-5x+3}{x^2-3}$?

(A) $x = \sqrt{3}$

(B) $y = 3$

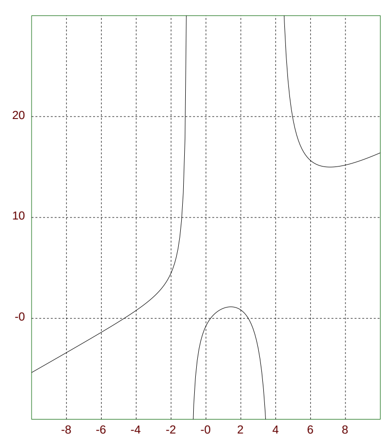
(C) $y = 2x + 3$

(D) None of these, but $f(x)$ does have an asymptote.

(E) $f(x)$ does not have any asymptotes.

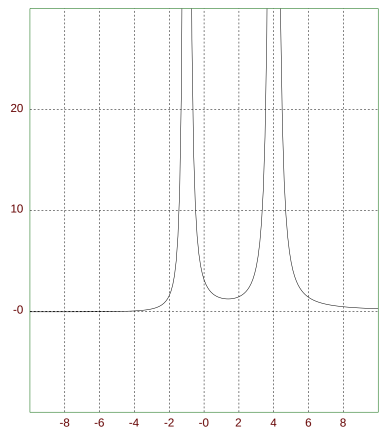
20. Let $f(x) = \frac{2x^3+10x^2+5x-5}{(x+1)^2(x-4)}$. It is possible to calculate $f'(x) = -\frac{14x^3+24x^2+45x+55}{(x+1)^3(x-4)^2}$ so $f'(x) = 0$ has a solution at -1.41... and $f''(x) = -\frac{28x^4+58x^3+252x^2+152x-370}{(x+1)^4(x-4)^3}$ so $f''(x) = 0$ has solutions at 0.86... and -1.6325...

Which of the following is the graph of $f(x)$?

(a) 

(b) 

(c)



(d)

(e)

21. Solve the system of equations

$$\begin{array}{rrrcr} a & +2b & -3c & = & 4 \\ 3a & +5b & -7c & = & 12 \\ 2a & +5b & -5c & = & 2 \end{array}$$

22. Given the following table, find a set of simultaneous equations that need to be solved in order to calculate the number of units of each sector that must be produced in order to meet the external demand of 20 units of sector A, 30 units of sector B and 30 units of sector C. [You do not need to solve the simultaneous equations.]

	Sector A	Sector B	Sector C	External Demand	Total
Sector A	100	300	150	450	1000
Sector B	300	600	150	150	1200
Sector C	50	150	150	400	750
Other costs	700	150	300		
Total	1000	1200	750		

23. Find the maximum of $f(x) = x^3 - x^2 + 7$ on the interval $[-1, 1]$. Justify why your answer must be the maximum.

24. Find all asymptotes to the function $f(x) = \frac{x^3-5x+3}{x^2+2x+4}$.

25. For two products, A and B, the demand functions for the products are given by:

$$q_A = 1000 - 3p_A - p_B \tag{1}$$

$$q_B = \frac{500}{p_A + 10p_B} \tag{2}$$

(a) Calculate the partial derivatives $\frac{\partial q_A}{\partial p_A}$, $\frac{\partial q_A}{\partial p_B}$, $\frac{\partial q_B}{\partial p_A}$, and $\frac{\partial q_B}{\partial p_B}$. Determine whether the products are competitive or complementary (or neither).

(b) Suppose the products are both produced by the same company. Now what equations should be solved to find the prices p_A and p_B that the company should charge to maximise its total revenue. [You do not need to solve these equations, but you should simplify them to equations which involve just p_A and p_B.]