MATH 1115, Mathematics for Commerce
WINTER 2011
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Homework Sheet 2
Due: Wednesday 2nd February: 2:30 PM

Each multiple choice question is worth one mark, other questions are worth two marks.

1. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & -5 \end{pmatrix}$, then $A^{\mathrm{T}} + 2B$ is:

   (A) $\begin{pmatrix} 3 & 8 \\ 7 & 12 \\ 5 & -5 \end{pmatrix}$

   (B) $\begin{pmatrix} 14 & 20 & -14 \\ 30 & 44 & -22 \\ 28 & 36 & -56 \end{pmatrix}$

   (C) $\begin{pmatrix} 3 & 7 & 5 \\ 8 & 12 & -5 \end{pmatrix}$

   (D) $\begin{pmatrix} 1 & 7 & 7 \\ 11 & 12 & -8 \end{pmatrix}$

   (E) undefined.

2. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & -5 \end{pmatrix}$ then the following matrices are defined:

   (A) AB, BA but not $AB + BA$.

   (B) AB but not BA.

   (C) BA but not AB

   (D) AB, BA and $AB + BA$

   (E) neither AB nor BA

3. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}$ then AB is

   (A) $\begin{pmatrix} 1 & 4 \\ 9 & -20 \end{pmatrix}$

   (B) $\begin{pmatrix} 7 & 15 \\ -8 & -14 \end{pmatrix}$

   (C) $\begin{pmatrix} 7 & -8 \\ 15 & -14 \end{pmatrix}$

(D) $\begin{pmatrix} 7 & 10 \\ -12 & -14 \end{pmatrix}$

(E) undefined.

4. If $A = \begin{pmatrix} 1 & 2 & 2 \\ 3 & 4 & 1 \end{pmatrix}$ then A^{-1} is

(A) $\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{4} & 1 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 4 \\ -1 & -3 \\ 1 & 1 \end{pmatrix}$

(D) defined, but not equal to (A), (B) or (C).

(E) undefined.

5. For the system of equations:

$$\begin{array}{rrrrr} x & +2y & -z & = & 4 \\ 2x & -y & & = & 3 \\ x & +y & -z & = & -1 \end{array}$$

the solution includes:

(A) $x = 3$

(B) $y = 5$

(C) $z = 7$

(D) There is no solution.

(E) There are infinitely many solutions.

6. The matrix $A = \begin{pmatrix} 18 & 5 & 8 & 5 & 1 & 0 & 1 \\ 5 & 20 & -12 & -4 & -3 & 0 & -1 \\ 8 & -12 & 16 & 9 & 4 & 1 & 1 \\ 5 & -4 & 9 & 13 & 6 & 4 & -2 \\ 1 & -3 & 4 & 6 & 3 & 2 & -1 \\ 0 & 0 & 1 & 4 & 2 & 2 & -1 \\ 1 & -1 & 1 & -2 & -1 & -1 & 1 \end{pmatrix}$ is invertible

with inverse $A^{-1} = \begin{pmatrix} 1 & -2 & -3 & 4 & -6 & 1 & 3 \\ -2 & 5 & 8 & -12 & 17 & -2 & -10 \\ -3 & 8 & 14 & -21 & 27 & -2 & -20 \\ 4 & -12 & -21 & 34 & -45 & 3 & 31 \\ -6 & 17 & 27 & -45 & 67 & -8 & -35 \\ 1 & -2 & -2 & 3 & -8 & 4 & 1 \\ 3 & -10 & -20 & 31 & -35 & 1 & 36 \end{pmatrix}$.

Which of the following is part of the solution to the system of equations

$$\begin{array}{rrrrrrrcr}
18a & +5b & +8c & +5d & +e & & +g & = & 2 \\
5a & +20b & -12c & -4d & -3e & & -g & = & 6 \\
8a & -12b & +16c & +9d & +4e & +f & +g & = & 3 \\
5a & -4b & +9c & +13d & +6e & +4f & -2g & = & 2 \\
a & -3b & +4c & +6d & +3e & +2f & -g & = & -1 \\
 & & c & +4d & +2e & +2f & -g & = & -2 \\
a & -b & +c & -2d & -e & -f & +g & = & 1
\end{array}$$

(A) $a = 5$

(B) $b = -6$

(C) $c = 4$

(D) $d = 11$

(E) $e = 10$

7. In an economy with three sectors, and whose Leontief matrix is $A = \begin{pmatrix} 0.2 & 0.3 & 0.6 \\ 0.3 & 0.1 & 0.3 \\ 0.1 & 0.4 & 0.2 \end{pmatrix}$ the amount of production needed in each sector to satisfy external demands $\begin{pmatrix} 30 \\ 15 \\ 25 \end{pmatrix}$ is:

(A) (150 100 100)

(B) (25.5 18 14)

(C) (55.5 33 39)

(D) (30 15 25)

(E) (43 35.5 52.5)

8. Solve the system of equations

$$\begin{array}{rrrrcr}
a & +2b & +3c & +d & = & 7 \\
2a & +3b & -c & +2d & = & 12 \\
a & -b & -15c & +d & = & 1 \\
3a & & -33c & +3d & = & 9
\end{array}$$

9. Given the following table, calculate the number of units of each sector that must be produced in order to meet the external demand of 20 units of sector A, 40 units of sector B and 35 units of sector C.

	Sector A	Sector B	Sector C	Other outputs	Total
Sector A	100	350	200	500	1150
Sector B	300	700	150	300	1450
Sector C	50	200	100	400	750
Other costs	700	200	300		
Total	1150	1450	750		