MATH 1115, Mathematics for Commerce
WINTER 2011
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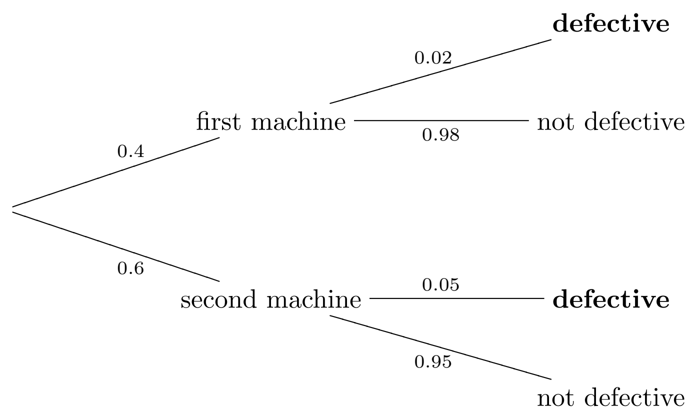
Homework Sheet 6
Model Solutions

1. *A company has two machines for producing a product. The first machine produces defective products 2% of the time. The second machine produces defective products 5% of the time. In a given year, the first machine produces 2,000 products, while the second machine produces 3,000. If a product produced by the company in that year is selected at random, the probability that it is defective is:*




The total probability that the product is defective is therefore $0.4 \times 0.02 + 0.6 \times 0.05 = 0.038$.

(A) 0.044

(B) 0.042

(C) 0.040

**(D) 0.038**

(E) 0.036

2. A pack of cards has two cards missing: the King of spades and the 3 of hearts. A card is drawn at random. What is the probability that it is a 7 conditional on it being a spade?

Let $A$ be the event that the card is a 7, and let $B$ be the event that it is a spade. Conditional on the event $B$, we can look at the reduced sample space consisting of all the spades, i.e. $\{2, 3, 4, 5, 6, 7, 8, 9, 10, \text{J,Q,A}\}$, which has 12 elements, one of which is a 7. The probability of a 7, conditional on the card being a spade is therefore $\frac{1}{12}$.

Alternatively, we can use the formula

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{50}}{\frac{12}{50}} = \frac{1}{12}$$

(A) $\frac{1}{50}$

(B) $\frac{4}{50}$

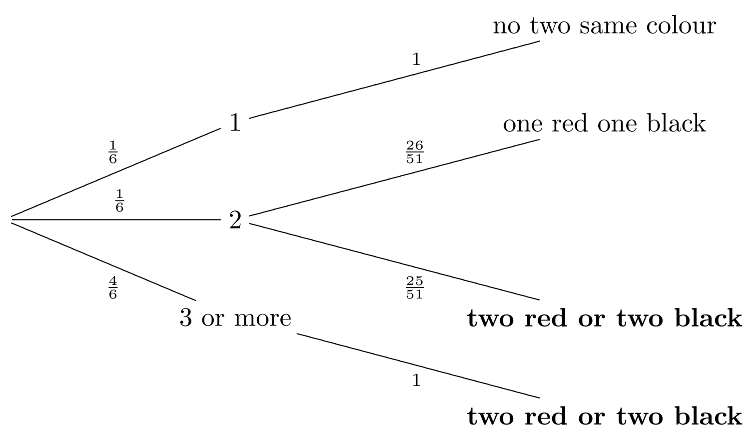
**(C)** $\mathbf{\frac{1}{12}}$

(D) $\frac{1}{4}$

(E) $\frac{12}{50}$

3. *A fair (6-sided) die is rolled. Then a number of cards equal to the number shown on the die are drawn from a standard deck of 52 cards (without replacement). The probability that the cards drawn include at least two of the same colour (red or black) is: [That is, the probability that there are either at least two red cards, or at least two black cards, or both. A standard deck has 26 red cards and 26 black cards.]*

   Clearly, if the number rolled is at least 3, then when we pick 3 or more cards, some two must be of the same colour. On the other hand, if the number rolled is 1, we can't have two cards of the same colour, as we only have one card. We therefore have the following probability tree.




   From this, we see that the probability of two red or two black cards is $\frac{25}{51\times 6} + \frac{4}{6}$.

**(A)** $\mathbf{\frac{25}{6\times 51} + \frac{4}{6}}$

(B) $\frac{25}{6\times 51} + \frac{25\times 24}{6\times 51\times 50} + \frac{25\times 24\times 23}{6\times 51\times 50\times 49} + \frac{25\times 24\times 23\times 22}{6\times 51\times 50\times 49\times 48}$

(C) $\frac{{}_{26}C_3}{{}_{52}C_3}$

(D) $\frac{3}{4}$

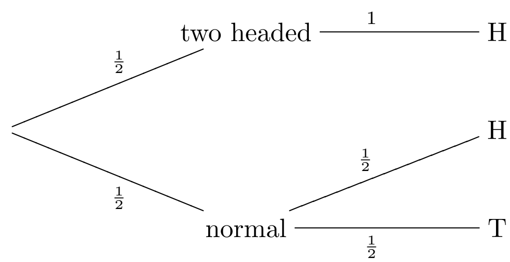
(E) $\frac{1}{2}$

4. *There are two coins in a wallet. One of them is a fair coin. The other has two heads. A coin is selected at random, and tossed. It comes down heads. The probability that it is the two-headed coin is:*

   We use Bayes formula for conditional probability. Let $A$ be the event that we pick the two-headed coin, and let $B$ be the event that we get a head when we toss the coin. Now, we want to use the formula

   $$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

   Drawing a probability tree:




   We see that $P(A \cap B) = \frac{1}{2}$, while $P(B) = \frac{3}{4}$. Therefore,

   $$P(A|B) = \frac{\left(\frac{1}{2}\right)}{\left(\frac{3}{4}\right)} = \frac{2}{3}$$

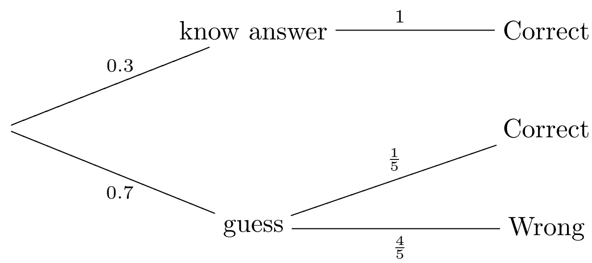
   (A) $\frac{1}{3}$
   (B) $\frac{1}{2}$
   **(C)** $\boldsymbol{\frac{2}{3}}$
   (D) $\frac{3}{4}$
   (E) 1

5. *In a multiple choice question with 5 options, assume that 30% of students know the answer to the question, and therefore get it right, and the remaining 70% guess an answer at random. If a certain student gets the question right, the probability that they actually know the answer is:*

   We use Bayes formula for conditional probability. Let $A$ be the event that the student knows the answer, and let $B$ be the event that the student gets the question correct. Now, we want to use the formula

   $$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

   Drawing a probability tree:




We see that $P(A \cap B) = 0.3$, while $P(B) = 0.3 + 0.7 \times 0.2 = 0.44$. Therefore,

$$P(A|B) = \frac{0.30}{0.44} = \frac{15}{22}$$

(A) $\frac{10}{27}$

(B) $\frac{3}{10}$

(C) $\frac{1}{2}$

**(D)** $\mathbf{\frac{15}{22}}$

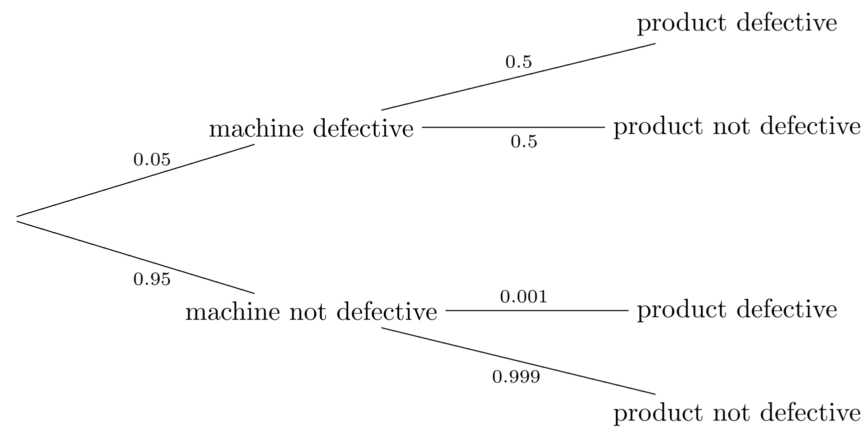
(E) $\frac{4}{5}$

6. *A company is performing quality control testing on one of its products, to determine whether the machine for manufacturing them needs to be replaced. The machine has probability 0.95 of being in working order. In this case, only 0.1% of the products it produces will be defective. On the other hand, if the machine is defective, 50% of the products it produces will be defective. A product produced by this machine is selected at random and found to be defective. What is the probability that the machine is defective?*

   We use Bayes formula for conditional probability. Let $A$ be the event that the machine is defective, and let $B$ be the event that the product is defective. Now, we want to use the formula

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

   Drawing a probability tree:



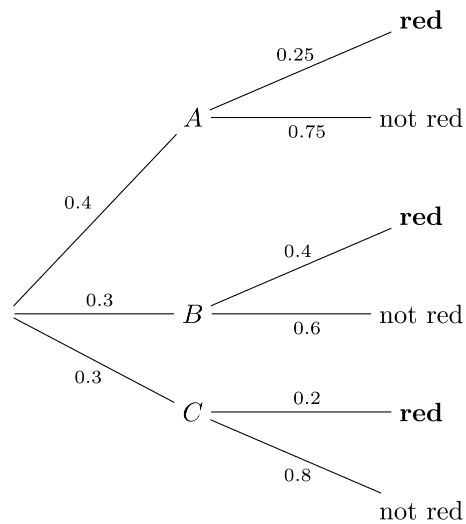

We see that $P(A \cap B) = 0.05 \times 0.5 = 0.025$, while $P(B) = 0.025 + 0.95 \times 0.001 = 0.02595$. Therefore,

$$P(A|B) = \frac{0.025}{0.02595} = \frac{500}{519}$$

(A) $\frac{1}{20}$
(B) $\frac{46}{537}$
(C) $\frac{1}{2}$
(D) $\frac{293}{478}$
**(E)** $\boldsymbol{\frac{500}{519}}$

7. *A company makes 3 different models of car, A, B and C. The cars come in 3 different colours: red, green and blue. Of the cars sold, 40% are model A, 30% are model B and 30% are model C. Of the model A cars, 25% are red, 50% are green and 25% are blue. Of the model B cars, 40% are red, 20% are green and 40% are blue. Of the model C cars, 20% are red, 50% are green and 30% are blue. What proportion of all cars made by the company are red?*

We draw the probability tree:



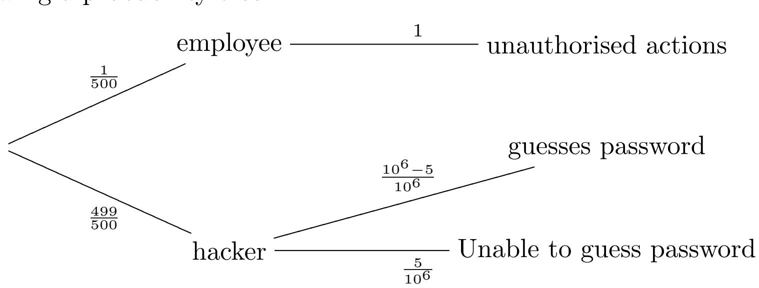

The overall probability of a red car is $0.4 \times 0.25 + 0.3 \times 0.4 + 0.3 \times 0.2 = 0.28$ [or 28%].

8. *A company is concerned about its computer security. Each employee has a password, which consists of any 6 digits 0–9. When an employee wants to use their account, they are given 5 chances to enter the correct password, before the account is closed for investigation. The company's IT department spot some unauthorised actions on one account. The account-holder claims that he was not responsible for the unauthorised actions, and that a hacker must have correctly guessed his password in order to abuse his account. The employee's manager estimates that the prior probability that this employee was responsible for the unauthorised actions on that account is 1 in 500. What is the probability that the employee was responsible?*

   We use Bayes formula for conditional probability. Let $A$ be the event that the employee is responsible, and let $B$ be the event that the unauthorised actions take place. Now, we want to use the formula

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

   Drawing a probability tree:



We see that $P(A \cap B) = \frac{1}{500}$, while $P(B) = \frac{1}{500} + \frac{499}{500} \times \frac{5}{1000000} = \frac{200499}{100000000}$. Therefore,

$$P(A|B) = \frac{\left(\frac{1}{500}\right)}{\left(\frac{200499}{100000000}\right)} = \frac{200000}{200499}$$

[This is approximately 0.9975]