MATH 2112/CSCI 2112, Discrete Structures I
Winter 2007
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Homework Sheet 5
Due in: Friday 16th February, 1:30 PM

Compulsory questions

1 Use Euclid's algorithm to find the greatest common divisor of the following pairs of numbers. Write down all the steps involved.

(a) 123,456 and 654,321

(b) 1,111,111 and 12,121,212

2 Find integers a and b such that $13579a + 2468b = 1$.

3 (a) Show that any number congruent to 3 modulo 4 is divisible by a prime number congruent to 3 modulo 4.[You may assume that the product of any collection of integers that are all congruent to 1 modulo 4 is also congruent to 1 modulo 4.]

(b) Prove that there are infinitely many prime numbers congruent to 3 modulo 4.

4 Are the following numbers rational or irrational? Give proofs:

(a) $\sqrt{6}$

(b) $\sqrt{2}+\sqrt{3}$ [Hint: What is $\left(\sqrt{2}+\sqrt{3}\right)^2$?]

5 Show that the difference between a rational number and an irrational number is irrational.

6 Observe that $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{\left(\sqrt{2}\times\sqrt{2}\right)} = \sqrt{2}^2 = 2$. Prove that there are two irrational numbers α and β such that α^β is rational.

8 Find $0 \leqslant n < 2310$ satisfying:

$$\begin{aligned} n &\equiv 7 \pmod{11} && (1)\\ n &\equiv 10 \pmod{14} && (2)\\ n &\equiv 11 \pmod{15} && (3) \end{aligned}$$

Bonus Question

7 Prove that if a positive integer n is not a square, then $\sqrt{n}$ is irrational.