MATH 2600/STAT 2600, Theory of Interest
FALL 2013
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Homework Sheet 4
Model Solutions

1. *Calculate the price that should be paid for each of the following bonds to obtain the desired yield:*

   (a) Face value \$100,000, maturing at par in 10 years, coupon rate $j_2 = 4\%$, desired yield $j_2 = 5\%$.

   We use Makeham's formula. $K = 100000 \times 1.025^{-20} = 61027.09$, and $P = 61027.09 + (100000 - 61027.09)\frac{0.02}{0.025} = 92205.42$.

   (b) Face value \$80,000, maturing at par in 10 years, coupon rate $j_2 = 7\%$, desired yield $j_2 = 5\%$.

   We use Makeham's formula. $K = 80000 \times 1.025^{-20} = 48821.68$, and $P = 48821.68 + (80000 - 48821.68)\frac{0.035}{0.025} = 92471.33$.

2. *At what interest rate would the two bonds in Question 1 have the same present value?*

   The difference between the two bonds is a semi-annual payment of \$800 for 10 years, and a payment of \$20,000 after 10 years (By the difference, I mean the net outcome of selling one bond and buying the other). For the two bonds to have the same price, this difference would have to have net value zero, so the accumulated value of the semi-annual payments of \$800 would have to be \$20,000. That is, we are looking for the solution to $s_{\overline{20}|i} = 25$. We try a few values:

i	$s_{\overline{20}\|i}$
2.5000%	25.545
2.0000%	24.297
2.3000%	25.037
2.2000%	24.787
2.2850%	24.999
2.2900%	25.012
2.2875%	25.005

   So their values are equal at interest rate $j_2 = 4.57\%$.

3. *(a) Write out a complete bond amortisation schedule for a bond with face value \$10,000 with coupon rate $j_2 = 2\%$, maturing at par in 5 years, sold to an investor who wishes to receive a yield of $j_2 = 7\%$*

   Using Makeham's formula, we have $K = 10000(1.035)^{-10} = 7089.19$, so $P = 7089.19 + (10000 - 7089.19)\frac{2}{7} = 7920.85$. At this price and yield, the

semiannual interest should be $7920.85 \times 0.035 = 277.23$, but the actual interest received is only \$100.

Outstanding Balance	Interest	Payment	Principal Repaid
7920.85	277.23	100	−177.23
8098.07	283.43	100	−183.43
8281.51	289.85	100	−189.85
8471.36	296.50	100	−196.50
8667.86	303.38	100	−203.38
8871.24	310.49	100	−210.49
9081.73	317.86	100	−217.86
9299.59	325.49	100	−225.49
9525.08	333.38	100	−233.38
9758.45	341.55	10100	9758.45

(b) Write out a complete bond amortisation schedule for a bond with face value \$10,000 with coupon rate $j_2 = 6\%$, maturing at par in 5 years, sold to an investor who wishes to receive a yield of $j_2 = 2.5\%$

Using Makeham's formula, we have $K = 10000(1.0125)^{-10} = 8831.81$, so $P = 8831.81 + (10000 - 8831.81)\frac{6}{2.5} = 11635.48$.

Outstanding Balance	Interest	Payment	Principal Repaid
11635.48	145.44	300	154.56
11480.91	143.51	300	156.49
11324.42	141.56	300	158.44
11165.98	139.57	300	160.43
11005.55	137.57	300	162.43
10843.12	135.54	300	164.46
10678.66	133.48	300	166.52
10512.14	131.40	300	168.60
10343.55	129.29	300	170.71
10172.84	127.16	10300	10172.84

4. *A bond has face value \$20,000, maturity in 10 years, coupon rate $j_2 = 4\%$. After 2 years and 2 months, it is sold to Mr. Zack, who wishes to receive a yield of 6%. Calculate*

   (a) The flat price.

   After 2 years, at $j_2 = 6\%$, we use Makeham's formula: $K = 20000 \times 1.03^{-16} = 12463.34$, so $P = 12463.34 + (20000 - 12463.34)\frac{4}{6} = 17487.78$. The price two months later is then given by $17487.78(1.03)^{\frac{1}{3}} = 17660.94$.

   (b) The quoted price.

   The next coupon has a value of \$400, and the seller is entitled to one third of this value. Therefore, the quoted price is obtained by subtracting \$133.33 from the flat price, to get \$17527.60.

5. *Mr. Allen buys a bond with face value \$6,000, maturing at par in 9 years, with coupon rate 2%, for a price to yield 4.7%. After two years, interest*

rates increase, and he sells the bond to an investor who wishes to receive a yield of 5.9%. What is Mr. Allen's rate of return?

We calculate the amount Mr. Allen pays using Makeham's formula: $K = 6000(1.0235)^{-18} = 3949.75$, so $P = 3949.75 + (6000 - 3949.75)\frac{2}{4.7} = 4822.20$. Similarly, using Makeham's formula for the sale two years later: $K = 6000(1.0295)^{-14} = 3993.76$, and $P = 3993.76 + (6000 - 3993.76)\frac{2}{5.9} = 4673.84$.

Mr. Allen therefore receives 4 coupons for \$60, and the payment of \$4673.84. We need to determine the interest rate at which this has a present value of \$4822.20. We try some values of j_2:

j_2	Present value
1.000%	\$4,818.55
0.700%	\$4,846.89
0.900%	\$4,827.98
0.960%	\$4,822.32
0.965%	\$4,821.85

So his yield is $j_2 = 0.96\%$.