

MATH 2600/STAT 2600, Theory of Interest

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Homework Sheet 5

Model Solutions

1. A company are considering a project. The project has the following expected cash flows (all amounts are calculated at the beginning of the year):

Year	0	1	2	3	4	5	6	7	8	9
Net Cash Flow (000)	-500	100	200	150	0	-100	300	200	300	100

(a) What is the Net Present value of the project at $j_1 = 4\%$?

At $j_1 = 4\%$, the net present value is $100(1.04)^{-9} + 300(1.04)^{-8} + 200(1.04)^{-7} + 300(1.04)^{-6} - 100(1.04)^{-5} + 150(1.04)^{-3} + 200(1.04)^{-2} + 100(1.04)^{-1} - 500 = 510.76549$.

(b) What is the Net Present value of the project at $j_1 = 9\%$?

At $j_1 = 9\%$, the net present value is $100(1.09)^{-9} + 300(1.09)^{-8} + 200(1.09)^{-7} + 300(1.09)^{-6} - 100(1.09)^{-5} + 150(1.09)^{-3} + 200(1.09)^{-2} + 100(1.09)^{-1} - 500 = 295.80321$.

(c) What is the internal rate of return?

(i) 11.54%

(ii) 15.28%

(iii) 18.29%

(iv) 20.82%

We calculate the NPV for each of these interest rates.

j_1	NPV
11.54%	212.12731
15.28%	111.76386
18.29%	46.45125
20.82%	-0.05023

So the internal rate of return is 20.82%.

2. A company are considering two projects. The projects have the following expected cash flows (all amounts are calculated at the beginning of the year):

Year	0	1	2	3	4	5	6
Project 1 Net Cash Flow (000)	-350	100	100	200	50	30	10
Project 2 Net Cash Flow (000)	-120	10	20	30	30	30	40

(a) Which project should they prefer if the cost of capital is $j_1 = 2\%$?

At $j_1 = 2\%$, the net present values are: for Project 1: $10(1.02)^{-6} + 30(1.02)^{-5} + 50(1.02)^{-4} + 200(1.02)^{-3} + 100(1.02)^{-2} + 100(1.02)^{-1} - 350 = 114.86447$; for Project 2: $40(1.02)^{-6} + 30(1.02)^{-5} + 30(1.02)^{-4} + 30(1.02)^{-3} + 20(1.02)^{-2} + 10(1.02)^{-1} - 120 = 27.70311$. Therefore, they should prefer the first project.

(b) Which project should they prefer if the cost of capital is $j_1 = 12\%$?

At $j_1 = 12\%$, the net present values are: for Project 1: $10(1.12)^{-6} + 30(1.12)^{-5} + 50(1.12)^{-4} + 200(1.12)^{-3} + 100(1.12)^{-2} + 100(1.12)^{-1} - 350 = 15.22617$; for Project 2: $40(1.12)^{-6} + 30(1.12)^{-5} + 30(1.12)^{-4} + 30(1.12)^{-3} + 20(1.12)^{-2} + 10(1.12)^{-1} - 120 = -17.42055$. Therefore, they should prefer the first project.

(c) Which project has the higher internal rate of return?

Project 1 has the higher internal rate of return, since its internal rate of return is more than 12%, while the internal rate of return of Project 2 is less than 12%. [The internal rates of return are 13.92% and 7.51% respectively.]

3. Mr. Zhang has \$250,000 in his investment fund at the start of the year. 2 months later, he withdraws \$110,000. Another 3 months later (5 months from the start of the year), he withdraws a further \$80,000 (there is enough money in the fund to cover this withdrawal). After another 3 months, (8 months from the start of the year), he inherits some money and invests \$300,000 in the fund. At the end of the year, there is \$410,000 in the fund. What is his Dollar-weighted rate of return for the year?

His dollar-weighted rate of return i satisfies $250000(1+i) - 110000 \left(1 + \frac{10}{12}i\right) - 80000 \left(1 + \frac{7}{12}i\right) + 300000 \left(1 + \frac{4}{12}i\right) = 410000$, which gives $\frac{2540000}{12}i = 50000$, so $i = \frac{600000}{2540000} = 23.62\%$.

4. Mr. Archibald is managing a pension fund. At the start of the year, there is \$8,000,000 in the fund. The fund activity is summarised in the following table:

Months from start of year	Fund value before transaction	Net Deposit	Fund value after transaction
0			8,000,000
2	7,820,000	-100,000	7,720,000
3	7,750,000	-400,000	7,350,000
5	7,220,000	-100,000	7,120,000
7	7,770,000	600,000	8,370,000
8	8,160,000	-200,000	7,860,000
10	7,990,000	100,000	8,090,000
12	7,920,000		7,920,000

Calculate his time-weighted rate of return for the year.

His time-weighted rate of return for the year was $\left(\frac{7820}{8000}\right) \left(\frac{7750}{7720}\right) \left(\frac{7220}{7350}\right) \left(\frac{7770}{7120}\right) \left(\frac{8160}{8370}\right) \left(\frac{7990}{7860}\right) \left(\frac{7920}{8090}\right) - 1 = 2.06\%$.

5. A company has an account which pays interest at $j_1 = 3.5\%$ on credit balances, and charges interest at $j_1 = 5\%$ on debit balances. The company is considering using this account to fund possible projects with the following expected cashflows: (All amounts are calculated at the beginning of the year.)

Year	0	1	2	3	4	5	6
Project 1 Net Cash Flow (000)	-150	10	20	30	50	60	30
Project 2 Net Cash Flow (000)	-180	60	60	40	40	20	10

How much money does the company have in its account at the end of 7 years if it invests in:

(a) The first project?

If the company invests in the first project, it has a negative balance for the first 5 years, after which time, its balance is $60 + 50(1.05) + 30(1.05)^2 + 20(1.05)^3 + 10(1.05)^4 - 150(1.05)^5 = -10.55967$, which is still negative, so after 6 years, it has a balance of $30 - 10.55967(1.05) = 18.91234$.

(b) The second project?

If the company invests in the second project, it has a negative balance for the first 4 years, after which it has a balance of $40 + 40(1.05) + 60(1.05)^2 + 60(1.05)^3 - 180(1.05)^5 = -1.183625$, which is still negative, so after 5 years, it has a balance of $20 - 1.183625(1.05) = 18.75719$. This is positive, so after 6 years, its balance is $18.75719(1.035) + 10 = 29.41370$.

(c) Both projects?

If it invests in both projects, then the first 5 years are as in (a) and (b), with balances all negative. After 5 years, its balance is $18.75719 - 10.55967 = 8.19752$, which is positive, so after 6 years it has a balance of $8.19752(1.035) + 40 = 48.48444$.