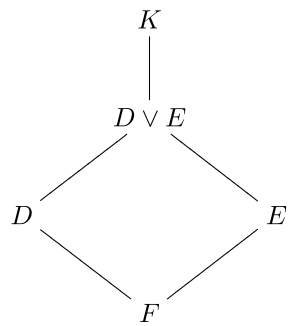
MATH 3030, Abstract Algebra
Winter 2013
Toby Kenney
Homework Sheet 16
Due: Wednesday 27th March: 3:30 PM

Basic Questions

1. Let f be an irreducible quartic (degree 4) polynomial over a perfect field F. Let K be a splitting field for f over F. Let the zeros of f in K be α, β, γ and δ.
   (a) What is the orbit of $\alpha\beta + \gamma\delta$ under $G(K/F)$?
   (b) [bonus] If $f(x) = x^4 + ax^3 + bx^2 + cx + d$, what is $\mathrm{Irr}(\alpha\beta + \gamma\delta, F)$?
2. Write $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$ as a rational function in the elementary symmetric functions $a + b + c$, $ab + ac + bc$ and abc.
3. What is the order of $G(GF(64)/GF(4))$?
4. How many extension fields of $\mathbb{Q}$ are contained in the field $\mathbb{Q}(\sqrt[4]{3}, i)$?

Theoretical Questions

5. Let E be a finite normal extension of F. Let $\alpha \in E$. Define the *norm* of α over F by:
$$N_{E/F}(\alpha) = \Pi_{\sigma \in G(E/F)} \sigma(\alpha)$$
and the *trace* of α over F by:
$$\mathrm{Tr}_{E/F}(\alpha) = \sum_{\sigma \in G(E/F)} \sigma(\alpha)$$
Show that $N_{E/F}(\alpha)$ and $\mathrm{Tr}_{E/F}(\alpha)$ are elements of F.
6. Let D and E be two extension fields of F. Let K be an extension field of F containing both D and E. The join $D \vee E$ of D and E is the smallest subfield of K that contains both D and E as subfields — see the following diagram:



Describe $G(K/(D \vee E))$ in terms of $G(K/D)$ and $G(K/E)$.

7. Let f be an irreducible monic polynomial over a field F, and let K be a splitting field for f over F. Let the zeros of f in K be $\alpha_1, \ldots, \alpha_n$. Let $\Delta(f) = \Pi_{i<j}(\alpha_i - \alpha_j)$. Show that $(\Delta(f))^2 \in F$.