MATH/STAT 3460, Intermediate Statistical Theory
Winter 2014

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Sample Midterm Examination

This Sample Midterm has more questions than the actual midterm, in order to cover a wider range of questions.

Basic Questions

1. The number of hours of sunlight on a given day of the year is believed to follow a Normal distribution with mean 6 and variance σ^2. Over a series of 10 years, the number of hours of sunlight observed on this day is 4.9, 6.8, 4.6, 8.7, 9.9, 2.3, 3.1, 6.4, 8.9, and 4.2. What is the maximum likelihood estimate for σ?

2. The remaining lifetime (in years) of a patient undergoing a certain kind of treatment is exponentially distributed with parameter λ. In a study which follows 10 patients for a period of 3 years, seven of the patients have lifetimes: 0.3, 0.8, 0.9, 1.4, 1.8, 2.5, and 2.9, while the remaining three patients survive to the end of the three-year period.

   (a) Show that $\frac{7}{19.6}$ is the maximum likelihood estimate for λ.

   (b) Show that $[0.14, 0.73]$ is a 10% likelihood interval for λ.

3. A team of doctors wants to determine how common a certain disease is. They test 1000 individuals at random, and find that 5 of them have the disease. Use a normal approximation to find a 10% likelihood interval for the probability that an individual has the disease.

4. The number of people visiting a doctor's office on a given day is believed to be a Poisson distribution with parameter $\sqrt[3]{a^2+5}$. Over a series of 10 days, the total number of people who visit the office is 841. What is the MLE of a?

5. The length of time between eruptions of a certain volcano, in years is believed to follow an exponential distribution with parameter λ. The following observations are made:

Length of time (years)	frequency
0–100	23
100–200	36
200–400	45
over 400	26

   Use Newton's method to find the MLE for λ. Start with an estimate of 0.003, and perform two steps.

6. In a trial for a new drug, the probability of a response to dose d is assumed to be $1 - \frac{1}{1+e^{\alpha+\beta d}}$ for some α and β. The data from a study of the drug are given in the following table:

dose	-1	0	1	2
number	24	26	23	21
number of responses	2	8	15	18

(a) Show that $\alpha = -0.85106$ and $\beta = 1.39655$ is the maximum likelihood estimate for α and β, and calculate the observed information matrix at these values.

(b) Use a normal approximation to calculate a 10% likelihood region for (α, β).

7. We observe two samples from a Poisson distribution with parameter λ. If the true value of λ is 1.3, what is the probability that this value lies within a 10% likelihood interval?

8. Let $X_1, \ldots, X_{100}$ be samples from a normal distribution with mean 0 and variance σ^2. If $\sum X_i{}^2 = 180$, show that $[1.175, 1.551]$ is a 95% confidence interval for σ. [You may use the chi-square approximation.]

9. The lifetime of a particle (in nanoseconds) is exponentially distributed with parameter λ. However, checking whether the particle has decayed will influence the lifetime. A scientist plans an experiment to keep 1000 particles for a fixed time period T, and after that time, to measure how many particles have decayed. Show that $T = \frac{1.5936}{\lambda}$ gives the highest expected information about λ.

10. Let $X_1, \ldots, X_{40}$ be samples from a uniform distribution on $[0, a]$. The MLE for a is $\max(X_1, \ldots, X_{40})$. What is the bias of this estimate? [Hint: the expected value of a positive random variable with cumulative distribution function $F(x)$ is $\int_0^\infty (1 - F(x))dx$.]