ACSC/STAT 3703, Actuarial Models I (Further Probability with Applications to Actuarial Science)
WINTER 2015

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Sample Final Examination

This Sample examination has more questions than the actual final, in order to cover a wider range of questions. Estimated times are provided after each question to help your preparation.

1. An insurance company models claim sizes as having the following survival function

$$S(x) = \frac{25(x+1)}{(x^2+2x+5)^2} \qquad x \geqslant 0$$

   Calculate the TVaR of a loss at the 95% level. [10 mins]

2. An insurance company models claim sizes as following a mixture of two distributions. With probability 0.3, claims follow a Weibull distribution with $\tau = 2$ and $\theta = 100$. With probability 0.7, claims follow a Pareto distribution with $\alpha = 3$ and $\theta = 250$.

   (a) Which of the following is the VaR of the distribution at the 90% level? [10 mins]

   (i) 122.02

   (ii) 146.35

   (iii) 197.14

   (iv) 230.60

   (b) Calculate the TVaR at the 90% level. [5 mins]

3. An insurance company observes the following sample of claims:

   1.2,1.6,2.1,3.5, 3.7

   They use a Kernel density model with a uniform kernel with bandwidth 1. What is the probability that a claim is between 2.3 and 3.3? [5 mins]

4. An insurance company observes the following sample of claims:

   2.3, 2.6, 2.8, 3.0, 3.5

   They use a Kernel density model with kernel a gamma distribution with $\alpha = 2$ and $\theta = \frac{x}{2}$ (so the mean matches the observed data point). What is the variance of a random claim under this model? [10 mins]

5. An insurance company observes the following sample of claims:

   1.6, 2.2, 2.4, 3.1, 3.5

   They use a Kernel density model with a uniform kernel with bandwidth b. They calculate the VaR at the 90% level is 3.7. What bandwidth are they using? [15 mins]

6. An insurance company observes the following sample of claims:

   1.4, 1.9, 2.8, 2.9, 3.6

   They use a Kernel density model with a Gaussian (normal) kernel with standard deviation 1. What is the mode claim size (highest probability density)? [10 mins]

   (i) 2.50238

   (ii) 2.69101

   (iii) 2.82243

   (iv) 2.94337

7. An insurance company assigns a risk factor Θ to each individual. These Θ follow a gamma distribution with $\alpha = 2$ and $\theta = 400$. For an individual with risk factor $\Theta = \theta$, the size of a claim follows an inverse gamma distribution with $\alpha = 3$ and this value of θ. What is the probability that a random individual makes a claim in excess of \$3,000? [10 mins]

8. An insurance company models the sizes of claims as a mixture distribution. Conditional on $\Theta = \theta$, the claim size has survival function

$$S(x) = \frac{\theta^4}{x^4} \qquad x \geqslant \theta$$

   The distribution of Θ is a gamma distribution with $\alpha = 4$ and $\Theta = 600$. For two claims which share a value of Θ, what is the probability that both are larger than \$7,000? [10 mins]

9. An insurance company models life expectancy as having hazard rate $\lambda(t) = \Theta e^{0.03t}$, where Θ varies between individuals following a normal distribution with $\mu = 0.0000013$ and $\sigma = 0.0000000021$. Calculate the probability that an individual aged 50 survives to age 80. [10 mins]

10. An insurance company models life expectancy as having hazard rate $\lambda(t) = \frac{\Theta}{120-t}$, where Θ is uniformly distributed on the interval $[0.05, 0.35]$. Calculate the expected future lifetime of an individual aged 60. [10 mins]

11. An insurance company models an individual's lifetime as having density function $f(x) = \begin{cases} 0.0000521566(12-x)^2 & \text{if } 0 < x < 10 \\ 0.36e^{0.06x-e^{0.06x}} & \text{if } x > 10 \end{cases}$ Calculate the probability that an individual dies between ages 6 and 66. [10 mins]

12. An insurance company models the loss on a given policy as following a gamma distribution with $\alpha = 3$ and $\theta = 1500$ for values less than 15000, and following a Pareto distribution with $\alpha = 3$ and $\theta = 1200$ for values greater than 15000. 8% of claims are more than 15000. Calculate the variance of a random claim. [15 mins]

13. The number of incidents in a year follows a Poisson distribution with $\lambda = 5$. The number of claims resulting from an incident follows a negative binomial distribution with $r = 0.1$ and $\beta = 3.4$. Calculate the probability that there are exactly 3 claims in a given year. [10 mins]

14. The number of policies sold in a year follows a binomial distribution with $n = 100000$ and $p = 0.002$. The number of claims resulting from each policy sold follows a Poisson distribution with $\lambda = 0.02$. Calculate the variance of the total number of claims in a year. [10 mins]

15. The number of fires follows a Poisson distribution with $\lambda = 5$, and the number of earthquakes follows a Poisson distribution with $\lambda = 0.4$. The number of claims resulting from a fire follows a negative binomial with $r = 1$ and $\beta = 1.2$. The number of claims resulting from an earthquake follows a Poisson distribution with $\lambda = 8$. Calculate the probability that there are exactly 2 claims in a given year. [15 mins]

16. An insurance company models the number of claims resulting from 1200 policies as following a compound Poisson-Poisson distribution with parameters 3 and 8. The following year, the company sells 1400 policies. What is the probability that there are exactly 2 claims the following year? [10 mins]

17. An insurance company models the number of claims resulting from 1500 policies as following a compound Poisson-Poisson distribution with parameters 5 and 2. How many policies should the company sell the following year in order to make the probability of receiving at least 2 claims at least 0.99? [15 mins]

    (i) 1423

    (ii) 1641

    (iii) 1950

    (iv) 2274

18. An insurance company models loss size as following a Pareto distribution with $\alpha = 4$ and $\theta = 6000$. The company introduces a deductible of \$1,000. Calculate the expected payment per claim after the deductible is introduced. [10 mins]

19. An insurance company models loss size as following a Weibull distribution with $\tau = 2$ and $\theta = 2000$. The company wants to introduce a deductible so that the expected payment per loss is \$1400. What deductible should it introduce? [10 mins]

20. An insurance company models loss size as following a log-logistic distribution distribution with $\gamma = 2$ and $\theta = 2000$. The company wants to introduce a deductible with loss elimination ratio 30%.

    (a) What deductible should it introduce? [10 mins]

    (b) In the following years, there is uniform inflation of 4% every year. How many years does it take until the deductible calculated in (a) gives a loss elimination ratio of less than 25%? [10 mins]

21. Losses follow a generalised Pareto distribution with $\alpha = 2$, $\tau = 3$, and $\theta = 3000$. An insurance company introduces a deductible of \$600. Calculate the loss elimination ratio of this deductible after inflation of 12%. [10 mins]

22. Losses follow an inverse Pareto distribution with $\tau = 4$ and $\theta = 6000$.

    (a) Calculate the expected payment per claim with a policy limit of \$1,000,000. [10 mins]

    (b) Calculate the expected payment per claim if there is 15% inflation (the policy limit remains at \$1,000,000.) [5 mins]

23. Losses follow an exponential distribution with $\theta = 7000$. There is a deductible of \$700, a policy limit of \$25,000 and coinsurance such that the insurance pays 80% of the claim after the policy limit and deductible have been applied. Calculate the expected payment per claim and the variance of the payment per claim. [15 mins]

24. Losses follow a Pareto distribution with $\alpha = 3$ and $\theta = 5000$. There is a deductible of \$1000. The insurance company wants to reduce the TVaR (per claim) for this policy at the 99.9% level to \$60,000. What policy limit should they set? [15 mins]

25. Losses follow a Weibull distribution with $\tau = 3$ and $\theta = 4000$. Loss frequency follows a Negative binomial distribution with $r = 6$ and $\beta = 4$. The insurance company wants to reduce the expected number of claims to 22. What deductible should it introduce in order to achieve this? [10 mins]

26. Losses follow a gamma distribution with $\alpha = 2$ and $\theta = 2000$. Loss frequency follows a Poisson distribution with $\lambda = 4$. If the company introduces a deductible of \$600, what is the probabilitiy that it receives more than 2 claims after the deductible? [10 mins]

27. Losses follow a log-logistic distribution with $\gamma = 2$ and $\theta = 6000$. Claim frequency with a deductible of \$2,000 is modelled as a negative binomial with $r = 5$ and $\beta = 2.6$. what would be the distribution of the claim frequency if the deductible were removed? [10 mins]