ACSC/STAT 3703, Actuarial Models I (Further Probability with Applications to Actuarial Science)
Winter 2015

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Homework Sheet 6

Due: Friday 27th March: 12:30 PM

Basic Questions

1. Calculate the VaR and TVaR at the 95% level of the following distribution:

$$f(x) = \begin{cases} \frac{6(5x^4 - x^5)}{3125} & \text{for } 0 < x < 5 \\ 0 & \text{otherwise} \end{cases}$$

2. An insurance company observes the following sample of claims:

   2.8, 2.9, 3.4, 3.9, 4.7, 5.4

   It uses these to construct a Kernel density model with a uniform kernel with bandwidth 1.

   (a) Under this model, what is the probability that a random claim is greater than 4?

   (b) Under this model, what is the median claim size?

Standard Questions

3. An insurance company models the aggregate losses on a portfolio as following a Pareto distribution with $\theta = 1000000$ and $\alpha = \frac{10000}{N}$ where N is the number of policies issued. The risk management division asks them to ensure that the TVaR at the 99% level on the portfolio is at most 100,000. How many policies can they issue while ensuring this condition holds.

   (i) 104

   (ii) 203

   (iii) 297

   (iv) 480

4. Losses follow a gamma distribution with $\alpha = 3$. The insurance company uses the standard deviation principle $r(X) = \mu + a\sigma$ and VaR to measure the risk. For this distribution, what value of a gives the same value of the risk as VaR at the 95% level? [You can calculate the percentiles of the gamma distribution using the qgamma function in `R`, or using an online calculator — Google "gamma quantile calculator".]

5. An insurance company uses a kernel density model with a Gaussian kernel with standard deviation 1. It observes the following sample of 5 claims:

   1.4, 2.6, 2.7, 3.5, 3.8

   It wants to check that the VaR at the 95% level is at most 5.2. It plans to collect one additional data point. Based on the Kernel density model, what is the largest value of the last data point which will give a VaR of at most 5.2?