ACSC/STAT 3703, Actuarial Models I

WINTER 2023

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Practice Midterm Examination

This Sample examination has more questions than the actual final, in order to cover a wider range of questions. Estimated times are provided after each question to help your preparation.

1. An individual has utility function $u(x) = \frac{x}{x+a}$ for some constant a. The individual has wealth \$42,000 and is willing to pay up to \$142 to insure against a loss of \$23,000 with probability 0.006. How much would she be willing to pay to insure against a loss of \$19,000 with probability 0.064? [15 mins]

2. An individual has wealth \$132,000. The individual has utility function of the form $u(x) = a\log(x) - \frac{1}{x}$ for some value of a.

   The individual elects not to purchase insurance to cover a loss of \$22,000 with probability 0.014 for a premium of \$322. What is the most the individual might be willing to pay for insurance covering a loss of \$18,000 with probability 0.01?

   (i) \$193.40

   (ii) \$208.09

   (iii) \$220.51

   (iv) \$232.00

3. Which of the following risks are insurable? For risks which are not insurable, explain why they are not insurable.

   (i) The risk that a car will need to be replaced in 20 years time.

   (ii) The risk that an individual is abducted by aliens.

   (iii) The risk that a planned holiday will need to be cancelled.

   (iv) The risk that an individual will feel ill.

   (v) The risk that a banknote will be torn.

   (vi) The risk to a professional sports team that its star player will be injured and unable to play.

   (vii) The risk that an individual is not selected for a particular job.

   (viii) The risk that all civilisations on earth will be destroyed.

4. A homeowner's house is valued at \$560,000. However, the home is insured only to a value of \$360,000. The insurer requires 80% coverage for full insurance. The home sustains \$6,000 of fire damage. The deductible is \$5,000, decreasing linearly to zero for losses of \$8,000. How much does the insurer reimburse?

5. A marine insurance policy includes a deductible of \$10,000, a policy limit of \$5,000,000 and co-insurance of 20% payable by the policyholder. If the co-insurance is applied before the policy limit, how much would the insurer reimburse for a loss of

(i) \$5,000

(ii) \$15,000

(iii) \$5,200,000

(iv) \$10,000,000

6. An auto insurance company uses an expected loss ratio of 0.81. In accident year 2014, the earned premiums were \$1,420,000. In 2014, the insurance company made a total of \$189,300 in loss payments for accident year 2014, a total of \$152,500 in 2015, and a total of \$239,600 in 2016. What loss reserves should the company hold for this accident year at the end of 2016.

7. The following table shows the cumulative losses (in thousands) on claims from one line of business of an insurance company over the past 6 years.

	Development year					
Accident year	0	1	2	3	4	5
2011	751	1,022	1,448	1,133	1,473	1,493
2012	1,337	1,297	1,460	1,537	1,679	
2013	1,250	1,624	1,815	1,860		
2014	1,325	1,512	1,685			
2015	1,471	1,536				
2016	2,036					

Using the average for calculating loss development factors, esimate the total reserve needed for payments to be made in 2018 using.

(a) The loss development triangle method

(b) The Bornhuetter-Fergusson method. The expected loss ratio is 0.76 and the earned premiums in each year are given in the following table:

Year	Earned Premiums (000's)
2011	1943
2012	2430
2013	2623
2014	2804
2015	3356
2016	3673

8. An insurance company starts a new line of insurance in 2016, and collects a total of \$1,900,000 in premiums that year, and the estimated incurred losses for accident year 2016 are \$1,384,000. Half of the premium payments are made at the beginning of the year, and the other half are uniformly distributed over the year. An actuary is using this data to estimate rates for premium year 2018. Claims are subject to 4% inflation per year. By what percentage should premiums increase from 2016 in order to achieve a loss ratio of 0.75? [15 mins]

9. In 2021, a home insurer collected \$22,480,000 in earned premiums, and paid \$19,380,000 in payments. There was a rate change on 1st May 2021. Before the rate change, the premium was \$790. After the rate change, the premium was \$830. Ignoring inflation, what should the new premium be to achieve an expense ratio of 20%? [15 mins]

10. An insurance company is calculating the premium for a new line of insurance it started in 2018. The new line of insurance started on 1st May 2018, and half of the policies started at that time. Due to an advertising campaign, the

rate of policy purchases in November and December was twice the rate for the months from May to October. The annual premium in 2018 was \$600. The total premiums collected in 2018 were \$1,200,000 and the total losses were \$462,000. Assuming losses are uniformly distributed throughout the year, annual inflation is 5%, and the expense ratio is 0.2, calculate the new premium for policy year 2020.

11. For a certain line of insurance, the loss amount per claim follows a Pareto distribution with $\alpha = 4$. If the policy has a deductible per loss set at 0.1θ and a policy limit set at 2θ, by how much will the expected payment per loss increase if there is inflation of 5%? [10 mins]

12. The random variable X has density function given by

$$f(x) = \frac{15}{4}x(1-x)^2(2-x), 0 \leqslant x \leqslant 2$$

    (a) calculate the hazard rate of X.

    (b) Calculate the kurtosis of X

13. Losses follow a Pareto distribution with $\alpha = 3$. How large can θ be if the insurance company wants to limit its Value at Risk at the 95% level to \$15,000?

14. Calculate the moment generating function of a sum of 5 independent beta random variables with parameters 3 and 2

15. Which distribution has a heavier tail: a gamma distribution with $\alpha = 4$ and $\theta = 400$, or a Weibull distribution with $\tau = 4$ and $\theta = 400$? [Use any reasonable method for comparing tail-weight.]

16. Recall that desirable coherence properties for measures of risk are:

    - Subadditivity
    - Monotonicity
    - Positive homogeneity
    - Translation invariance

    Which properties are satisfied by the risk measure given by the measure $r(X) = \mu + \pi_{0.9}$ (the mean plus the 90th percentile)?

17. Calculate the TVaR of a gamma distribution with $\alpha = 3$ and $\theta = 2000$ at the 0.99 level. [The VaR at the 0.99 level is 16,811.894]

18. Claims follow a Pareto distribution with $\alpha = 4$. There is a policy limit which is currently exceeded by 0.16% of claims. There is uniform inflation of 8% per year on claim amounts. What proportion of claims will exceed the policy limit in 4 years time? [The policy limit does not change in these 4 years.]

19. You observe the following sample of insurance losses:

    `1.6 3.6 3.8 4.2 5.6`

    Using a Kernel density model with Gaussian (normal) kernel with standard deviation 1.2, estimate the probability that a loss exceeds 5.5.

20. You observe the following sample of insurance losses:

    1.6 3.6 3.8 4.2 5.6

Using a Kernel density model with triangular kernel with bandwidth 2, estimate the probability that a loss exceeds 5.5. A triangular kernel with bandwidth b centred at x_0 is given by the density function

$$f(x) = \begin{cases} \frac{x+b-x_0}{b^2} & \text{if } x_0 - b < x < x_0 \\ \frac{x_0+b-x}{b^2} & \text{if } x_0 < x < x_0 + b \\ 0 & \text{otherwise} \end{cases}$$

[5 mins]