ACSC/STAT 3703, Actuarial Models I

WINTER 2023
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Homework Sheet 3

Due: Wednesday 8th February: 11:30

Note: This homework assignment is only valid for WINTER 2023. If you find this homework in a different term, please contact me to find the correct homework sheet.

Basic Questions

1. A distribution has survival function $S(x) = \frac{3}{(2+x)^2} + \frac{16}{(4+x)^3}$ for $x \geqslant 0$. Calculate its hazard-rate.

2. A continuous random variable has moment generating function given by $M(t) = \frac{1}{(1-2t)^3(1-5t)^2}$ for $t < 0.2$. Calculate its coefficient of variation.

3. Calculate the mean excess loss function for a distribution with survival function given by $S(x) = \frac{2}{(x+1)^3} - \frac{32}{(x+2)^5}$ for $x \geqslant 0$.

4. Calculate the probability generating function of a discrete distribution with p.m.f. given by

$$f(x) = \frac{x^2}{6 \times 2^x}$$

for $n \geqslant 0$.

[We can show this is a probability mass function as follows:

We need to show

$$\sum_{n=0}^{\infty} \frac{n^2}{2^n} = 6$$

To do this, we combine the nth and $n+1$th terms in the series

$$\sum_{n=0}^{\infty} \frac{n^2}{2^n} = \frac{1}{2}\sum_{n=0}^{\infty} \left(\frac{n^2}{2^n} + \frac{(n+1)^2}{2^{n+1}}\right) + \frac{1}{2}\frac{0^2}{2^0}$$

The term in the sum can be rearranged to give

$$\sum_{n=0}^{\infty} \frac{n^2}{2^n} = \frac{1}{2}\sum_{n=0}^{\infty} \frac{\frac{3}{2}n^2 + n + \frac{1}{2}}{2^n} = \frac{3}{4}\sum_{n=0}^{\infty} \frac{n^2}{2^n} + \frac{1}{2}\sum_{n=0}^{\infty} \frac{n}{2^n} + \frac{1}{4}\sum_{n=0}^{\infty} \frac{1}{2^n}$$

A similar argument gives

$$\sum_{n=0}^{\infty}\frac{n}{2^n}=\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{n}{2^n}+\frac{n+1}{2^{n+1}}\right)+\frac{1}{2}\frac{0}{2^0}=\frac{1}{2}\sum_{n=0}^{\infty}\frac{\frac{3}{2}n+\frac{1}{2}}{2^n}=\frac{3}{4}\sum_{n=0}^{\infty}\frac{n}{2^n}+\frac{1}{4}\sum_{n=0}^{\infty}\frac{1}{2^n}$$

We know that $\sum_{n=0}^{\infty}\frac{1}{2^n}=2$, so we can solve these to get

$$\sum_{n=0}^{\infty}\frac{n}{2^n}=2$$

and

$$\sum_{n=0}^{\infty}\frac{n^2}{2^n}=6$$

You may need to do a similar derivation to get the probability generating function.]

Standard Questions

5. The total cost of handling a claim is $X+Y$ where X is a discrete non-negative random variable with probability generating function $P_X(z)=e^{-4(1-(2.6-1.6z)^{-2})}$ and Y is a continuous non-negative random variable with moment generating function $M_Y(t)=\left(0.6+\frac{0.4}{(1-t)^2}\right)^3$. X and Y are independant. What is the moment generating function of the total cost?

6. An insurance company is trying to fit an inverse Pareto distribution to its claims data. The survival function for this distribution is given by

$$S(x)=1-\frac{x^\tau}{(x+\theta)^\tau}$$

The insurance company wants to select α and θ so that the the 5th percentile and the 95th percentile match the observed values of 458 and 86,322 respectively. Which of the following values should they choose for τ and what should be the corresponding value be for θ?

(i) 1.467008

(ii) 1.882693

(iii) 2.898321

(iv) 4.405930

Bonus Question

7. For a particular infectious disease, the number of distinct uninfected people, N, infected by a single infected person has a distribution with probability generating function $P(z) = 1 - \sqrt{\frac{1-z}{2}}$.

   A pandemic begins with a single infected person, and the numbers of people infected by different people are independent.

   (a) What is the probability that the pandemic ever dies out (i.e. that only a finite number of total infections happen)?

   (b) What is the "probability generating function" for the total number of people infected? [Technically, since there is non-zero probability that this number is infinite, it is not a probability generating function.]