ACSC/STAT 3703, Actuarial Models I

WINTER 2023
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Homework Sheet 6

Due: Wednesday 22nd March: 11:30

Note: This homework assignment is only valid for WINTER 2023. If you find this homework in a different term, please contact me to find the correct homework sheet.

Basic Questions

1. Let X follow a negative binomial distribution with $r = 5.2$ and $\beta = 0.9$. What is the probability that $X = 6$?

2. The number of claims on each insurance policy over a given time period is observed as follows:

Number of claims	Number of policies
0	398
1	363
2	228
3	118
4	40
5 or more	13

   Which distribution(s) from the $(a, b, 0)$-class and $(a, b, 1)$-class appear most appropriate for modelling this data?

3. X follows an extended modified negative binomial distribution with $r = -0.5$ and $\beta = 1.2$, and $p_0 = 0.3$. What is $P(X = 5)$?

4. Let X follow a mixed negative binomial distribution with $\beta = 2.6$ and r following a gamma distribution with $\alpha = 4$ and $\theta = 3$. What is the probability that $X = 2$?

Standard Questions

5. An insurance company finds that claim frequency for an individual has mean 0.23 and variance 0.48. They consider modelling this using either a negative binomial distribution or a zero-inflated Poisson distribution. Which of these has a higher probability that the number of claims is 3 or more?

6. If the distribution of X is from the $(a, b, 1)$-class and $P(X = 2) = 0.04$ and $P(X = 4) = 0.09$, what is the largest possible value of $P(X = 3)$?

7. (a) Substituting the recurrence $p_n = \left(a + \frac{b}{n}\right) p_{n-1}$ for $n \geqslant 2$ into the PGF $P(z) = \sum_{n=0}^{\infty} p_n z^n$ and its derivatives, write down a differential equation satisfied by $P(z)$.

   (b) Show that the PGF of a distribution from the $(a, b, 1)$ class is

$$P(z) = \frac{(1 - p_0)\left(\frac{1-az}{1-a}\right)^{-\frac{a+b}{a}} + p_0 - (1-a)^{\frac{a+b}{a}}}{1 - (1-a)^{\frac{a+b}{a}}}$$

Bonus Question

8. Let X be a truncated Poisson distribution with $\lambda = 2$. Is there a non-zero discrete random variable Y independent of X such that $X + Y - 1$ is from the $(a, b, 1)$ family?

   [Hint: Use the convolution formula to determine the probability mass function for $X + Y - 1$, and apply the recurrence for the $(a, b, 1)$ class to get a recursive formula for $P(Y = n)$. You then just need to show that this recurrence gives a probability mass function.]