ACSC/STAT 3703, Actuarial Models I

WINTER 2023

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Homework Sheet 2

Model Solutions

Basic Questions

1. *An insurer collects \$9,360,000 in earned premiums for accident year 2022. The total loss payments are \$7,791,000. Payments are subject to inflation of 5%, and policies are sold uniformly throughout the year. If the insurer's permissible loss ratio is 80%, by how much should the premium be changed for policy year 2024?*

The loss ratio in 2022 is $\frac{7791000}{9360000} = 0.832371794872$. Without inflation, the premium should be adjusted by a factor of $\frac{0.832371794872}{0.8} = 1.04046474359$. Inflation from the start of 2022 to a random claim in accident year 2022 is

$$\int_0^1 (1.05)^t\,dt = \left[\frac{(1.05)^t}{\log(1.05)}\right]_0^1 = \frac{0.05}{\log(1.05)} = 1.02479671572$$

Inflation from the start of 2024 to a random claim time for policy year 2024 is

$$\begin{aligned}\int_0^1 t(1.05)^t\,dt + \int_1^2 (2-t)(1.05)^t\,dt &= \left(\frac{1.05}{\log(1.05)} - \frac{0.05}{\log(1.05)^2}\right) + 1.05\int_0^1 (1-t)(1.05)^t\,dt\\ &= \left(\frac{1.05}{\log(1.05)} - \frac{0.05}{\log(1.05)^2}\right) + 1.05\left(\int_0^1 1(1.05)^t\,dt - \int_0^1 t(1.05)^t\,dt\right)\\ &= 1.05\left(\frac{0.05}{\log(1.05}\right) - 0.05\left(\frac{1.05}{\log(1.05)} - \frac{0.05}{\log(1.05)^2}\right) \qquad = \frac{0.05^2}{\log(1.05)^2}\\ &= 1.05020830854\end{aligned}$$

Therefore, the premium should be adjusted by a factor

$$\frac{1.04046474359 \times 1.05^2 \times 1.05020830854}{1.02479671572} = 1.17555699938$$

This is an increase of 17.56%.

2. *An insurer is reviewing claims for a certain line of insurance from Accident year 2022. The earned premiums in 2022 were \$5.9 million. The base premium in 2022 was \$730. However there was a rate change from the old premium of \$680 on 1st September 2021, and that old premium still applied to some policies in force in 2022. The total losses in Accident Year 2022 were \$5.14 million. What should the new premium for Policy Year 2024 be if the permissible loss ratio is 0.75 and annual inflation is 6%?*

   [Assume policies are sold and losses occur uniformly through the year.]

   We first adjust the earned premiums to the current premium. The rate change happened 4 months before the end of 2021, so the old premium applied to $\frac{1}{2}\times\left(\frac{8}{12}\right)^2=\frac{2}{9}$ of policy-years in accident year 2022. Therefore, the adjusted earned premiums are $5.9\times\frac{730}{\frac{2}{9}\times 680+\frac{7}{9}\times 730}=\$5.99119010819$ million. The loss ratio is therefore $\frac{5.14}{5.99119010819}=0.857926373088$, so without inflation, the premiums should be adjusted by a factor $\frac{0.857926373088}{0.75}=1.14390183078$.

   Inflation from the start of 2022 to a random loss time in accident year 2022 is

$$\int_0^1 (1.06)^t\,dt=\left[\frac{1.06^t}{\log(1.06)}\right]_0^1=\frac{0.06}{\log(1.06)}=1.02970867194$$

   Inflation from the start of 2024 to a random loss time in Policy year 2024 is

$$\begin{aligned}\int_0^1 t(1.06)^t\,dt+\int_1^2(2-t)(1.06)^t\,dt&=\int_0^1 t(1.06)^t\,dt+1.06\int_0^1(1-t)(1.06)^t\,dt1.06\int_0^1(1.06)^t\,dt-0.06\int_0^1 t(1.06)^t\,dt\\&=1.06\frac{0.06}{\log(1.06)}-0.06\left(\left[\frac{t1.06^t}{\log(1.06)}\right]_0^1-\int_0^1\frac{(1.06)^t}{\log(1.06)}\right)\\&=\frac{0.06^2}{\log(1.06)^2}\\&=1.06029994907\end{aligned}$$

   The premium for policy year 2024 is therefore

$$730\times 1.14390183078\times 1.06^2\times\frac{1.06029994907}{1.02970867194}=\$966.13$$

3. *An insurance company has two lines of coverage in its Fire Insurance packages, with different expected loss ratios, and has the following data on recent claims:*

Policy Type	*Policy Year*	*Earned Premiums*	*Expected Loss Ratio*	*Losses paid to date*
	2020	*\$13,800,000*	*0.76*	*\$7,700,000*
Fire	*2021*	*\$14,400,000*	*0.75*	*\$4,600,000*
	2022	*\$15,500,000*	*0.76*	*\$2,500,000*
	2020	*\$6,500,000*	*0.74*	*\$2,100,000*
Earthquake	*2021*	*\$5,800,000*	*0.74*	*\$1,100,000*
	2022	*\$7,100,000*	*0.75*	*\$800,000*

Calculate the loss reserves at the end of 2022.

We calculate the expected losses and the expected unpaid losses.

Policy Type	Policy Year	Expected total Losses	Losses paid to date	Reserves Needed
	2020	\$10,488,000	\$7,700,000	\$2,788,000
Fire	2021	\$10,800,000	\$4,600,000	\$6,200,000
	2022	\$11,780,000	\$2,500,000	\$9,280,000
	2020	\$4,810,000	\$2,100,000	\$2,710,000
Earthquake	2021	\$4,292,000	\$1,100,000	\$3,192,000
	2022	\$5,325,000	\$ 800,000	\$4,525,000
Total				\$28,695,000

So the total loss reserves needed at the end of 2022 are \$28,695,000.

4. *The following table shows the cumulative paid losses (in thousands) on claims from one line of business of an insurance company over the past 5 years.*

Accident year	*Earned premiums*	*Development year 0*	*1*	*2*	*3*	*4*
2018	*15371*	*2483*	*6837*	*10591*	*12728*	*13267*
2019	*14506*	*3038*	*8319*	*10009*	*10031*	
2020	*25385*	*5379*	*11893*	*14621*		
2021	*6468*	*1940*	*3339*			
2022	*22870*	*5146*				

Assume that all payments on claims arising from accidents in 2018 have now been settled. Estimate the future payments arising each year from open claims arising from accidents in each calendar year using

(a) The loss development triangle method using mean loss development factors

First we compute the loss development factors:

0/1 $\frac{30388}{12840} = 2.36666666667$
1/2 $\frac{35221}{27049} = 1.30211837776$
2/3 $\frac{22759}{20600} = 1.10480582524$
3/4 $\frac{13267}{12728} = 1.04234758014$

Using these values to complete the table gives the following cumulative losses:

Accident year	0	1	2	3	4
LDF		2.36666666667	1.30211837776	1.10480582524	1.04234758014
2019	3038	8319.00	10009.000	10031.000	10455.79
2020	5379	11893.00	14621.000	16153.366	16837.42
2021	1940	3339.00	4347.773	4803.445	5006.86
2022	5146	12178.87	15858.326	17520.371	18262.32

The future payments are the differences between consecutive years:

Accident year	0	1	2	3	4
2019					425
2020				1532	684
2021			1009	456	203
2022		7033	3679	1662	742

(b) The Bornhuetter-Ferguson method with expected loss ratio 0.77, using average loss development factors.

The average LDFs are:

0/1	$\frac{1}{4}\left(\frac{6837}{2483}+\frac{8319}{3038}+\frac{11893}{5379}+\frac{3339}{1940}\right)=2.35599460686$
1/2	$\frac{1}{3}\left(\frac{10591}{6837}+\frac{10009}{8319}+\frac{14621}{11893}\right)=1.32719975772$
2/3	$\frac{1}{2}\left(\frac{12728}{10591}+\frac{10031}{10009}\right)=1.10198655692$
3/4	$\frac{13267}{12728}=1.04234758014$

This gives the following proportions of losses paid.

Development Year	Cumulative proportion of losses paid	Proportion of losses paid
0	$\frac{1}{1.04234758014\times1.10198655692\times1.32719975772\times2.35599460686}=0.27842008081$	0.27842008081
1	$\frac{1}{1.04234758014\times1.10198655692\times1.32719975772}=0.65595620883$	0.37753612802
2	$\frac{1}{1.04234758014\times1.10198655692}=0.870584921435$	0.214628712605
3	$\frac{1}{1.04234758014}=0.959372880077$	0.088787958642
4	1	0.040627119923

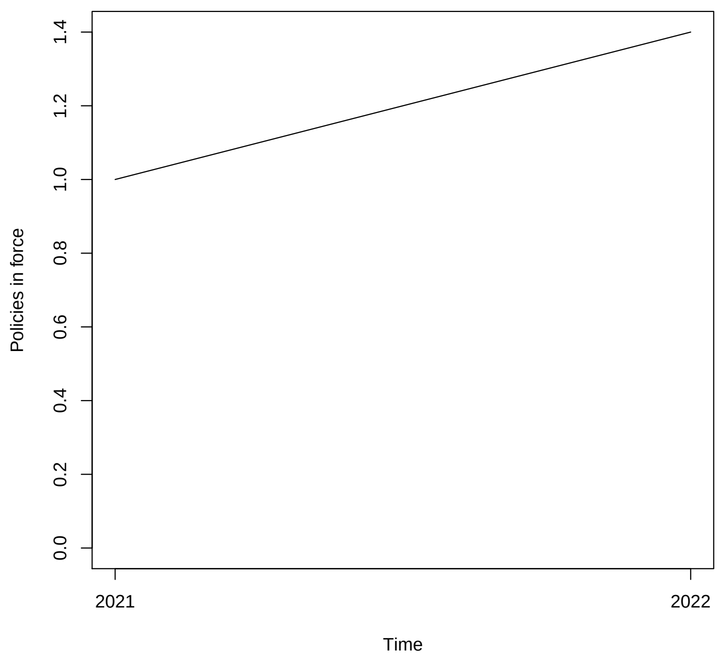
This gives the following reserves for mean LDF:

Accident year	Earned premiums	Expected Total claims	0	1	2	3	4
2019	14506	11169.62					453.7895
2020	25385	19546.45				1735.4894	794.1160
2021	6468	4980.36			1068.928	442.1960	202.3377
2021	22870	17609.90		6648.373	3779.590	1563.5471	715.4395

Standard Questions

5. *An insurance company is reviewing a line of insurance for accident year 2021. It finds that by increasing its premium by 8%, it would have achieved the desired loss ratio. The actuary estimates inflation will be 6%. Policies were sold uniformly during 2020, and were sold uniformly at a 40% higher rate in 2021 (that is, in any month of 2021, 1.4 times as many policies were sold as the same month in 2020). By how much should the premiums increase for policy year 2023, assuming policies are sold uniformly during 2023?*

   The number of policies in force at the end of 2021 is 1.4 times the number of policies at the start, and since policies are bought and expire at constant rates, the number of policies is a linear function. Thus, the number of policies in force at time t in 2021 is proportional to $1 + 0.4t$.



Thus the average inflation in accident year 2021 is

$$\begin{aligned}
\frac{1}{1.2}\left(\int_0^1 (1+0.4t)(1.06)^t\,dt\right) &= \frac{1}{1.2}\left(\left[\frac{1.06^t}{\log(1.06)}\right]_0^1 + 0.4\left[t\frac{1.06^t}{\log(1.06)}\right]_0^1 - 0.4\int_0^1 \frac{1.06^t}{\log(1.06)}\,dt\right)\\
&= \frac{1}{1.2}\left(\frac{0.06}{\log(1.06)} + 0.4\frac{1.06}{\log(1.06)} - 0.4\frac{0.06}{\log(1.06)^2}\right)\\
&= \frac{1.23765029314}{1.2}\\
&= 1.03137524428
\end{aligned}$$

The inflation from the start of 2023 to a random claim in policy year 2023 is $\frac{0.06^2}{\log(1.06)^2} = 1.06029994908$

Thus the premium needs to be increased by a factor $1.08\times 1.06^2\times\frac{1.06029994908}{1.03137524428} =$ 1.24752001926 or 24.75%.

6. *An insurance company has the following cumulative aggregate loss development data:*

Accident year	*Earned premiums*	*Development year 0*	*1*	*2*	*3*	*4*
2018	*21832*	*3908*	*7216*	*8596*	*14688*	*16939*
2019	*26322*	*5904*	*8011*	*12717*	*20535*	
2020	*16472*	*8002*	*11066*	*14044*		
2021	*27447*	*4315*	*11526*			
2022	*41419*	*8659*				

From this table, it calculates the following mean loss development factors:

Development year	*LDF*
0/1	*1.709024*
1/2	*1.344731*
2/3	*1.652653*
3/4	*1.153254*

and the following cumulative reserves:

Accident year	*Development year 0*	*1*	*2*	*3*	*4*
2019					*23682.08*
2020				*23209.86*	*26766.88*
2021			*15499.36*	*25615.08*	*29540.70*
2022		*14798.44*	*19899.92*	*32887.66*	*37927.84*

An adjustment to a previously closed claim means that the cumulative losses for 2021, development year 1 should have been $7,390.

(a) By how much the the necessary reserves at the end of 2022 decrease? [These are the total reserves for all expected payments after 2022 from all accident years.]

Changing the losses for 2021, development year 1 to \$7,390 changes the 0/1 LDF to $\frac{33683}{22129} = 1.52212029464$. With this new LDF, the total reserves needed for accident year 2022 are

$$8659 \times 1.52212029464 \times 1.344731 \times 1.652653 \times 1.153254 = 33779.9326097$$

and the total reserves needed for accident year 2021 are

$$7390 \times 1.344731 \times 1.652653 \times 1.153254 = 18940.284625$$

Thus the total reserves needed for accident year 2021 are $18940.284625 - 7390 = 11550.284625$ instead of $29540.70 - 11526 = 18014.70$, and reserves for accident year 2022 are $33779.9326097 - 8659 = 25120.9326097$ instead of $37927.84 - 8659 = 29268.84$. Thus the reserves needed are reduced by $18014.70 + 29268.84 - 11550.284625 - 25120.9326097 = 10612.3227653$

(b) Using the Bornhuetter-Fergusson method with expected loss ratio 0.81, the reserves for each year are:

Accident	*Expected*	*Development year*				
year	*Claims*	*0*	*1*	*2*	*3*	*4*
2019	*21320.82*					*2833.294*
2020	*13342.32*				*4568.851*	*1773.042*
2021	*22232.07*			*2990.319*	*7612.996*	*2954.389*
2022	*33549.39*		*5430.695*	*4512.552*	*11488.420*	*4458.331*

How much will the total reserves be changed if the cumulative losses for 2021, development year 1 are changed to \$7,390.

Replacing the LDF for 0/1 by the corrected LDF, 1.52212029464, the proportion of total losses in each year is given by

Development Year	Cumulative proportion of losses paid	Proportion of losses paid
0	$\frac{1}{1.52212029464 \times 1.344731 \times 1.652653 \times 1.153254} = 0.256335620916$	0.256335620916
1	$\frac{1}{1.344731 \times 1.652653 \times 1.153254} = 0.390173650835$	0.133838029919
2	$\frac{1}{1.652653 \times 1.153254} = 0.524678603661$	0.134504952826
3	$\frac{1}{1.153254} = 0.867111668375$	0.342433064714
4	1	0.132888331625

Accident	Expected	Development year				
year	Claims	0	1	2	3	4
2019	21320.82					2833.294
2020	13342.32				4568.851	1773.042
2021	22232.07			2990.319	7612.996	2954.389
2022	33549.39		4490.184	4512.552	11488.420	4458.331

Thus, the total reserves needed are

$$2833.294 + 4568.851 + 1773.042 + 2990.319 + 7612.996 + 2954.389 + 4490.184 + 4512.552 + 11488.420 + 4458.331 = 47682.378$$

Before the change, the reserves needed were

$$2833.294 + 4568.851 + 1773.042 + 2990.319 + 7612.996 + 2954.389 + 5430.695 + 4512.552 + 11488.420 + 4458.331 = 48622.889$$

so the reserves are reduced by $48622.889 - 47682.378 = \$940.511$.