ACSC/STAT 3720, Life Contingencies I

Winter 2015

Toby Kenney

Homework Sheet 7

Model Solutions

Basic Questions

1. *An insurance company sells 400 whole life insurance policies with annual premiums and benefits to select lives aged 36. The death benefit on these policies is \$400,000. The interest rate is $i = 0.08$. In the first year of the policies:*

   - No policyholders die.
   - The company earns interest $i = 0.07$.

   The company still uses $i = 0.08$ as its basis for calculating the policy values. What is the company's annual profit on these policies? [Using the lifetable in Table 1, we have $A_{[36]} = 0.0251133$ and $A_{[36]+1} = 0.0268981$.]

   We calculate $\ddot{a}_{[36]} = \frac{1-0.0251133}{\frac{0.08}{1.08}} = 13.160970$. The premium on the policies is therefore $\frac{400000\times0.0251133}{13.160970} = \$763.27$. We calculate $\ddot{a}_{[36]+1} = \frac{1-0.0268981}{\frac{0.08}{1.08}} = 13.136876$. The policy value after 1 year is $400000 \times 0.0268981 - 763.27 \times 13.136876 = 732.26$. After 1 year, the company has $400 \times 763.27 \times 1.07 = \$326,679.56$ and has liabilities of $400 \times 732.26 = 292902.77$, so its profit is $326679.56 - 292902.77 = \$33,776.79$.

2. *An insurance company sells 500 whole-life insurance policies to select lives aged 53. The death benefit of these policies is \$300,000. The interest rate is $i = 0.05$ and premiums are payable annually in advance. At this interest rate, $A_{[53]} = 0.165754$. In the first two years of the policy:*

   - one policyholder dies in the second year of the policy.
   - The company earns interest $i = 0.06$ in the first year of the policy, and $i = 0.04$ in the second year.

   Calculate the asset share of the remaining policies after the second year.

   We calculate $\ddot{a}_{53} = 21(1 - 0.165754) = 17.519164$. The premiums are $\frac{300000\times0.165754}{17.519164} = \$2,838.39$. The company receives premiums of 2838.39×500 in each year, so the accumulated value of these premiums is $2838.39 \times 500 \times (1.06 \times 1.04 + 1.04) = \$3,040,483.73$. When the benefits of \$300,000 for the policyholder who dies are subtracted the remaining balance is \$2,740,483.73. This is divided between the remaining 499 policyholders, leaving an asset share of \$5,491.95 per policy.

3. *A select life aged 27 purchases a whole-life insurance policy with a death benefit of \$1,000,000. The interest rate is $i = 0.06$. From the lifetable in Table 1, we have $A_{[27]} = 0.0324095$ and $A_{[27]+5} = 0.0423772$. Using Woolhouse's formula:*

   (a) calculate the monthly premium.

$\ddot{a}_{[27]} = \frac{1-0.0324095}{\left(\frac{0.06}{1.06}\right)} = 17.0940988$, and from Table 1, $\mu_{[27]} \approx \frac{1}{2}\left(q_{[26]} + q_{[27]}\right) = \frac{1}{2}\left(\frac{1.17}{9997.00} + \frac{1.32}{9995.14}\right) = 0.000124549646863$, and force of interest is $\delta = \log(1.06) = 0.05826891$, so Woolhouse's formula gives $\ddot{a}^{(12)}_{[27]} = 17.0940988 - \frac{11}{24} - \frac{143}{1728}(0.00012454964686 + 0.05826891) = 16.630933$. This gives $A^{(12)}_{[27]} = 1 - d^{(12)}\ddot{a}^{(12)}_{[27]} = 0.0285770$, so the monthly premium is $\frac{1000000\times 0.0285770}{12\times 16.630933} =$ \$143.19.

(b) calculate the policy value after 4 years and 3 months.

We calculate $\ddot{a}_{[27]+5} = \frac{1-0.0423772}{\left(\frac{0.06}{1.06}\right)} = 16.9180028$, so using Woolhouse's formula with $\mu_x = -\frac{1}{2}\log\left(\frac{9983.18}{9988.24}\right) = 0.0002533621$ and $\delta = \log(1.06) = 0.05826891$, we get $\ddot{a}^{(12)}_{[27]+5} = 16.9180028 - \frac{11}{24} - \frac{143}{1728}(0.0002533621 + 0.05826891) = 16.4548265$. Now if we assume uniform distribution of deaths, then on the lifetable, we have $l_{[27]+4.25} = \frac{1}{4}9985.80 + \frac{3}{4}9988.24 = 9987.63$, so the probability that an individual aged 31 and 3 months survives to age 32 is $\frac{9985.80}{9987.63}$. We have

$$\ddot{a}^{(12)}_{[27]+4.25} = \frac{9985.80}{9987.63}\ddot{a}^{(12)}_{[27]+4}(1.06)^{-\frac{9}{12}} +$$
$$\frac{1.83}{9987.63}\left(1 + \frac{8}{9}1.06^{-\frac{1}{12}} + \frac{7}{9}1.06^{-\frac{2}{12}} + \frac{6}{9}1.06^{-\frac{3}{12}} + \frac{5}{9}1.06^{-\frac{4}{12}} + \frac{4}{9}1.06^{-\frac{5}{12}} + \frac{3}{9}1.06^{-\frac{6}{12}} + \frac{2}{9}1.06^{-\frac{7}{12}} + \frac{1}{9}1.06^{-\frac{9}{12}}\right)$$
$$= 16.480267$$

We calculate $A^{(12)}_{[27]+4.25} = 1 - d^{(12)}\ddot{a}^{(12)}_{[27]+4.25} = 0.0419525$, so the policy value is $1000000 \times 0.0419525 - 143.19 \times 12 \times 16.480267 = 13634.76$.

(c) calculate the policy value after 4 years 2.2 months.

After 4 years 2.2 months, we have $l_{[27]+4.18333333} = 9987.79$, so the probability of surviving to the end of the month is $\frac{9987.63}{9987.79} = 0.999983980$, so the policy value is $(0.999983980 \times 13634.76 + 0.00001601 \times 1000000)1.06^{-\frac{0.8}{12}} = \$13,597.64$.

Standard Questions

4. *An insurance company is designing a new 10-year term insurance policy with continuous cash flows. The company wants the premium to be at a constant rate of P per year. The company wants the policy value to be given by ${}_tV = c - a(5-t)^2$, for some positive value a and c. If mortality follows the Gompertz law $\mu_x = AB^x$, use Thiele's differential equation to find the death benefit which achieves this.*

Thiele's differential equation states that

$$\frac{d}{dt}{}_tV = {}_tV\delta + P - \mu_{x+t}D_{x+t}$$

Substituting ${}_tV = c - a(5-t)^2$ gives

$$2a(5-t) = (c - a(5-t)^2) + P - AB^{x+t}D_{x+t}$$

which gives

$$D_{x+t} = \frac{a((6-t)^2 - 1) - c + P}{AB^{x+t}}$$

[To get the policy value to equal zero at the start and end of the policy, we need to set $c = 25a$.]

5. *An insurance company is valuing its policies. It finds that the total value of a large group of 300 policies was $3,200,000. These policies all had a death benefit of $1,200,000. The total annual premium for all these policies is $84,000. The interest rate is* $i = 0.05$. *150 of the policies have a mortality rate* $q_x = 0.00014$, *100 have a mortality rate* $q_x = 0.00025$ *and the remaining 50 have a mortality rate* $q_x = 0.0004$. *If there are no expenses associated with the policies, and none of the policy holders dies, what is the total value of all these policies the following year?*

   After receiving the premiums, the value was $3,284,000, and after interest, the value increased to 3448200. The expected mortality on the policies was $150 \times 0.00014 + 100 \times 0.00025 + 50 \times 0.0004 = 0.115$, so the expected death benefits paid are $1200000 \times 0.115 = 138000$. The expected value of the policies is $3448200 - 138000 = 3310200$, and the number of policies is larger than expected by a factor of $\frac{300}{299.885}$, so the total value of the policies is $\frac{300}{299.885} \times 3310200 = \$3,311,469.40$.

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$	x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66	74	8987.73	8932.10	8862.49	8775.52
26	9997.00	9995.83	9994.40	9992.66	75	8897.04	8836.71	8761.27	8667.10
27	9995.14	9993.90	9992.38	9990.52	76	8798.69	8733.34	8651.66	8549.78
28	9993.16	9991.84	9990.22	9988.24	77	8692.13	8621.41	8533.09	8423.00
29	9991.05	9989.65	9987.92	9985.80	78	8576.81	8500.36	8404.95	8286.16
30	9988.81	9987.30	9985.46	9983.18	79	8452.13	8369.60	8266.68	8138.66
31	9986.40	9984.80	9982.82	9980.38	80	8317.52	8228.53	8117.67	7979.93
32	9983.83	9982.11	9979.99	9977.37	81	8172.36	8076.57	7957.35	7809.41
33	9981.07	9979.23	9976.95	9974.13	82	8016.08	7913.13	7785.15	7626.56
34	9978.11	9976.13	9973.68	9970.64	83	7848.11	7737.67	7600.54	7430.89
35	9974.93	9972.79	9970.16	9966.88	84	7667.89	7549.66	7403.05	7221.99
36	9971.50	9969.20	9966.36	9962.82	85	7474.92	7348.64	7192.27	6999.51
37	9967.80	9965.33	9962.25	9958.44	86	7268.77	7134.21	6967.86	6763.22
38	9963.81	9961.14	9957.82	9953.69	87	7049.07	6906.07	6729.62	6513.04
39	9959.50	9956.61	9953.02	9948.55	88	6815.55	6664.05	6477.46	6249.02
40	9954.84	9951.71	9947.82	9942.98	89	6568.09	6408.10	6211.48	5971.42
41	9949.79	9946.41	9942.19	9936.94	90	6306.70	6138.35	5931.96	5680.73
42	9944.32	9940.66	9936.08	9930.38	91	6031.59	5855.15	5639.41	5377.67
43	9938.39	9934.41	9929.45	9923.26	92	5743.19	5559.08	5334.61	5063.27
44	9931.96	9927.64	9922.25	9915.52	93	5442.15	5250.97	5018.61	4738.86
45	9924.97	9920.28	9914.42	9907.10	94	5129.44	4931.97	4692.79	4406.12
46	9917.37	9912.28	9905.91	9897.94	95	4806.33	4603.54	4358.89	4067.08
47	9909.11	9903.58	9896.65	9887.98	96	4474.39	4267.51	4018.96	3724.10
48	9900.13	9894.11	9886.57	9877.13	97	4135.60	3926.04	3675.44	3379.91
49	9890.36	9883.80	9875.59	9865.30	98	3792.25	3581.66	3331.11	3037.57
50	9879.71	9872.57	9863.63	9852.42	99	3447.02	3237.23	2989.05	2700.39
51	9868.12	9860.34	9850.59	9838.38	100	3102.90	2895.94	2652.63	2371.88
52	9855.48	9847.01	9836.39	9823.08	101	2763.19	2561.21	2325.37	2055.64
53	9841.72	9832.48	9820.90	9806.39	102	2431.39	2236.61	2010.90	1755.27
54	9826.71	9816.64	9804.02	9788.18	103	2111.15	1925.80	1712.81	1474.18
55	9810.34	9799.37	9785.60	9768.33	104	1806.12	1632.34	1434.48	1215.44
56	9792.49	9780.52	9765.51	9746.67	105	1519.82	1359.55	1178.94	981.65
57	9773.03	9759.97	9743.60	9723.05	106	1255.46	1110.36	948.70	774.71
58	9751.79	9737.56	9719.69	9697.28	107	1015.81	887.14	745.58	595.71
59	9728.63	9713.10	9693.62	9669.17	108	802.96	691.49	570.56	444.87
60	9703.36	9686.43	9665.17	9638.51	109	618.23	524.17	423.71	321.41
61	9675.80	9657.33	9634.15	9605.07	110	462.04	385.00	304.13	223.65
62	9645.73	9625.59	9600.31	9568.61	111	333.80	272.80	210.00	149.10
63	9612.94	9590.98	9563.42	9528.85	112	231.99	185.53	138.71	94.62
64	9577.18	9553.24	9523.19	9485.52	113	154.19	120.34	87.07	56.74
65	9538.19	9512.09	9479.35	9438.30	114	97.30	73.90	51.50	31.84
66	9495.69	9467.25	9431.58	9386.86	115	57.78	42.55	28.41	16.52
67	9449.37	9418.39	9379.54	9330.85	116	31.92	22.69	14.43	7.81
68	9398.90	9365.17	9322.87	9269.88	117	16.15	11.04	6.63	3.30
69	9343.95	9307.23	9261.20	9203.55	118	7.34	4.79	2.69	1.21
70	9284.12	9244.18	9194.11	9131.43	119	2.90	1.79	0.93	0.37
71	9219.03	9175.59	9121.17	9053.07	120	0.95	0.55	0.26	0.09
72	9148.24	9101.03	9041.91	8967.97	121	0.23	0.13	0.05	0.01
73	9071.30	9020.03	8955.85	8875.63	122	0.03	0.02	0.01	0.00