ACSC/STAT 3720, Life Contingencies I
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Homework Sheet 8
Model Solutions

Basic Questions

1. *A woman aged 36, who is a select life on Table 1 buys a 10-year term insurance policy with a death benefit of \$600,000. (The policy uses a net annual premium.) Five years later, she wants to surrender the policy. The interest rate is $i = 0.02$. If the insurance company pays a cash surrender value of 80% of the policy value, how much does she receive?*

   $A_{[36]:\overline{10}|} = 0.820648$, so $A^1_{[36]:\overline{10}|} = 0.820648 - \frac{9923.26}{9971.50}(1.02)^{-10} = 0.00426865$. $\ddot{a}_{[36]:\overline{10}|} = 51(1 - 0.820648) = 9.14693754$. The net premium for this policy is therefore \$280.01. After 5 years, we have $A_{41:\overline{5}|} = 0.905835$, so $A^1_{41:\overline{5}|} = 0.905835 - \frac{9923.26}{9953.69}(1.02)^{-5} = 0.00287315$, and $\ddot{a}_{41:\overline{5}|} = 51(1 - 0.905835) = 4.802415$. The policy value is therefore $600000 \times 0.00287315 - 280.01 \times 4.802415 = \$379.167$. The surrender value is $0.8 \times 379.167 = \$303.33$.

2. *A man aged 46 buys mortgage insurance on a mortgage for \$200,000 at $i = 0.04$ with annual repayments of \$13,839.77, for 22 years. The mortgage insurance pays off the outstanding balance on this mortgage at the time the individual dies. The individual is a select life on Table 1. The interest rate used to value the insurance contract is $i = 0.04$. The annual premiums for this mortgage insurance are therefore \$193.58. Two years later, the man sells the house and wants to cancel the policy. Use the retrospective method to calculate the policy value at that time.*

   The accumulated value of the premiums paid is $193.58(1.04^2 + 1.04) = 410.70$. The expected accumulated value of the death benefits is

   $$\frac{5.09 \times (200000 \times 1.04) \times 1.04 + 6.37 \times (200000(1.04)^2 - 13839.77 \times 1.04)}{9917.37} = 240.72$$

   So the expected accumulated value of the policy is $410.70 - 240.72 = \$169.98$. This however needs to be divided between the $\frac{9905.91}{9917.37}$ policyholders who survive, leaving a total retrospective value of $\frac{169.98 \times 9917.37}{9905.91} = \$170.17$.

   [The premium was calculated as follows:

   The mortgage repayments are $\frac{200000 \times 0.04}{1 - 1.04^{-22}} = \$13,839.77$. The outstanding balance after n years is therefore

   $$200000(1.04)^n - (1.04)13839.77\frac{1.04^{n-1} - 1}{0.04} = 359834.02 - 145994.25(1.04)^n$$

   . At $i = 0.04$, $A_{[46]:\overline{22}|} = 0.429156$, while at $i = 0$, $A_{[46]:\overline{22}|} = 1$, so the EPV of the benefit of the policy is $359834.02 \times 0.429156 - 145994.25 = 8430.67$. This is actually the EPV of a policy which always makes the final payment even if the man survives. To get the actual EPV, we see that the final payment is $359834.02 - 145994.25(1.04)^{22} = 13839.50$, so the actual EPV is $8430.67 - \frac{9438.30}{9917.37}13839.50(1.04)^{-22} = 2873.11$. We also get $\ddot{a}_{[46]:\overline{22}|} = 26(1 - 0.454867) = 14.841945$, so the premium is $\frac{2873.11}{14.841945} = \$193.58$]

3. *A man aged 53, who is a select life on Table 1 buys a 10-year endowment insurance with a benefit of \$700,000. The interest rate is* $i = 0.08$*, which gives* $A_{[53]} = 0.0729141$*,* $A_{[53]+1} = 0.0778819$*,* $A_{[53]+4} = 0.0937116$ *and* $A_{[53]+10} = 0.132398$*. Using a Full preliminary term of 1 year, calculate the policy value after 4 years.*

   Since $A_{[53]+1} = 0.0778819$, we have $\ddot{a}_{[53]+1} = 13.5(1 - 0.0778819) = 12.44859435$. We also have $A_{[53]+1:\overline{9}|} = A_{[53]+1} + (1.08)^{-9}_{9}p_{[53]+1}(1 - A_{[53]+10}) = 0.0778819 + (1.08)^{-9}\frac{9638.51}{9832.48}(1 - 0.132398) = 0.5033368$. This gives $\ddot{a}_{[53]+1:\overline{9}|} = 13.5(1 - 0.5033368) = 6.7049526$.

   This means the premium for the last 9 years of the policy is $\frac{700000\times0.5033368}{6.7049526} = \$52,548.59$. After 4 years, we have $A_{[53]+4:\overline{6}|} = A_{[53]+4} + (1.08)^{-6}_{6}p_{[53]+4}(1 - A_{[53]+10}) = 0.0937116 + (1.08)^{-6}\frac{9638.51}{9788.18}(1 - 0.132398) = 0.6320879$, and therefore $\ddot{a}_{[53]+4:\overline{6}|} = 13.5(1 - 0.6320879) = 4.966813$, so the policy value is $700000 \times 0.6320879 - 52548.59 \times 4.966813 = \$181,462.55$.

Standard Questions

4. *A woman aged 43, who is a select life on Table 1 buys a 10-year term insurance policy with a death benefit of \$500,000. The interest rate is* $i = 0.05$*, so* $A_{[43]:\overline{10}|} = 0.614981$*. Five years later, she wants to convert the policy to a whole life insurance. If the insurance company pays a cash surrender value of 85% of the policy value, and the woman goes through the underwriting process again, so that she is a select life at age 48, what is the new premium for the whole life insurance policy? [*$A_{[48]} = 0.133980$*.]*

   $A_{[43]:\overline{10}|} = 0.614981$, so $A^{1}_{[43]:\overline{10}|} = 0.614981 - \frac{9852.42}{9938.39}(1.05)^{-10} = 0.00637824$, and $\ddot{a}_{[43]:\overline{10}|} = 21(1 - 0.614981) = 8.0853997$, so the premium is $\frac{500000\times0.00637824}{8.0853997} = \$394.43$. After 5 years, we have $A_{48:\overline{5}|} = 0.783941$, so $A^{1}_{48:\overline{5}|} = 0.783941 - \frac{9852.42}{9907.10}(1.05)^{-5} = 0.00473933$, and $\ddot{a}_{48:\overline{5}|} = 21(1-0.783941) = 4.537239$, so the policy value is $500000\times0.00473933 - 4.537239\times394.43 = \$580.04$. The surrender value is \$493.04.

   We calculate $A_{[48]} = 0.133980423446089$, so $\ddot{a}_{[48]} = 18.186411$. The premium for the new life insurance policy is therefore $500000 \times 0.133980423446089 - 493.0418.186411 = \$3,656.42$.

5. *A man bought a whole life insurance policy 4 years ago. At the time, his age was 47, and he was rated a select life following Table 1. The benefit of the policy was \$800,000. The interest rate is* $i = 0.05$*. He now wants to convert the policy to a paid-up term policy with the same death benefit. The insurance company offers a cash surrender value of 85% of the policy value. What is the term of the new insurance contract? [*$A_{[47]} = 0.128315$*,* $A_{51} = 0.153031$*]*

   $A_{[47]} = 0.128315$, $\ddot{a}_{[47]} = 18.305380815904644$, so the premium was $\$5,607.76$.

   4 years later, we have $A_{51} = 0.153031$, and $\ddot{a}_{51} = 17.786349$, so the policy value is $800000 \times 0.153031 - 5,607.76 \times 17.786349 = \$22,683.22$. The cash surrender value is $0.85 \times 22683.22 = \$19,280.74$. This is the EPV of the benefit of the paid-up term policy. This gives $A^{1}_{51:\overline{t}|} = \frac{19280.74}{800000} = 0.024100925$. We have that $A^{1}_{51:\overline{t}|} = A_{51} - 1.05^{-t}{}_{t}p_{51}A_{51+t}$, so we need to solve $1.05^{-t}{}_{t}p_{51}A_{51+t} = 0.128930$.

   We try different values of t [We can calculate this by the recurrence $1.04^{-(t+1)}{}_{t+1}p_{51}A_{51+t+1} = 1.04^{-t}{}_{t}p_{51}A_{51+t} - 1.04^{-(t+1)}{}_{t}p_{51}q_{51+t}$

t	A_{51+t}	$A_{51+t}(1.04)^{-t}{}_tp_{51}$
0	0.153031	0.153031
1	0.159677	0.151891
2	0.166573	0.150359
3	0.173724	0.148766
4	0.181136	0.147113
5	0.188814	0.145394
6	0.196764	0.143610
7	0.20499	0.141756
8	0.213496	0.139829
9	0.222286	0.137828
10	0.231363	0.135750
11	0.24073	0.133592
12	0.25039	0.131350
13	0.260343	0.129024
14	0.270592	0.126609
15	0.281134	0.124104

So the new term is 13 years.

Bonus Question

6. *A woman aged 29 bought a 10-year term insurance with annual premiums. At the time, she was a select life from Table 1. If she is in good health 5 years later, she would be able to surrender her current policy and use the money to purchase a 5-year term insurance for the same death benefit. Since she is now a select life, she would benefit from a lower premium. What cash surrender value (as a percentage of policy value) should the insurance company offer her, so that this option results in the same premiums as her current policy? The current interest rate is $i = 0.06$.*

$\ddot{A}^1_{[29]:\overline{10}|} = 0.00197864$ and $a_{[29]:\overline{10}} = 7.794610$, so the premium (for a death benefit of \$1) is $\frac{0.00197864}{7.794610} = 0.000253847$. Five years later, if she is in good health, then we have $\ddot{A}^1_{[34]:\overline{5}|} = 0.00126442$ and $a_{[34]:\overline{5}|} = 4.462997$, [so the new premium would be $\frac{0.00126442}{4.462997} = 0.000283312$] To make the new premium equal to the current premium, the surrender value needs to be the difference in EPV, that is, $0.00126442 - 4.462997 \times 0.000253847 = 0.000131502$. The policy value of the current policy is based on the assumption that the woman is not a select life, so we get $A^1_{34:\overline{5}|} = 0.00146959$ and $\ddot{a}_{34:\overline{5}|} = 4.462370$, so the policy value is $0.00146959 - 4.462370 \times 0.000253847 = .000336832$, so the percentage of the current policy value that should be given as a cash surrender value is $\frac{0.000131502}{0.000336832} = 39.04\%$

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$	x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66	74	8987.73	8932.10	8862.49	8775.52
26	9997.00	9995.83	9994.40	9992.66	75	8897.04	8836.71	8761.27	8667.10
27	9995.14	9993.90	9992.38	9990.52	76	8798.69	8733.34	8651.66	8549.78
28	9993.16	9991.84	9990.22	9988.24	77	8692.13	8621.41	8533.09	8423.00
29	9991.05	9989.65	9987.92	9985.80	78	8576.81	8500.36	8404.95	8286.16
30	9988.81	9987.30	9985.46	9983.18	79	8452.13	8369.60	8266.68	8138.66
31	9986.40	9984.80	9982.82	9980.38	80	8317.52	8228.53	8117.67	7979.93
32	9983.83	9982.11	9979.99	9977.37	81	8172.36	8076.57	7957.35	7809.41
33	9981.07	9979.23	9976.95	9974.13	82	8016.08	7913.13	7785.15	7626.56
34	9978.11	9976.13	9973.68	9970.64	83	7848.11	7737.67	7600.54	7430.89
35	9974.93	9972.79	9970.16	9966.88	84	7667.89	7549.66	7403.05	7221.99
36	9971.50	9969.20	9966.36	9962.82	85	7474.92	7348.64	7192.27	6999.51
37	9967.80	9965.33	9962.25	9958.44	86	7268.77	7134.21	6967.86	6763.22
38	9963.81	9961.14	9957.82	9953.69	87	7049.07	6906.07	6729.62	6513.04
39	9959.50	9956.61	9953.02	9948.55	88	6815.55	6664.05	6477.46	6249.02
40	9954.84	9951.71	9947.82	9942.98	89	6568.09	6408.10	6211.48	5971.42
41	9949.79	9946.41	9942.19	9936.94	90	6306.70	6138.35	5931.96	5680.73
42	9944.32	9940.66	9936.08	9930.38	91	6031.59	5855.15	5639.41	5377.67
43	9938.39	9934.41	9929.45	9923.26	92	5743.19	5559.08	5334.61	5063.27
44	9931.96	9927.64	9922.25	9915.52	93	5442.15	5250.97	5018.61	4738.86
45	9924.97	9920.28	9914.42	9907.10	94	5129.44	4931.97	4692.79	4406.12
46	9917.37	9912.28	9905.91	9897.94	95	4806.33	4603.54	4358.89	4067.08
47	9909.11	9903.58	9896.65	9887.98	96	4474.39	4267.51	4018.96	3724.10
48	9900.13	9894.11	9886.57	9877.13	97	4135.60	3926.04	3675.44	3379.91
49	9890.36	9883.80	9875.59	9865.30	98	3792.25	3581.66	3331.11	3037.57
50	9879.71	9872.57	9863.63	9852.42	99	3447.02	3237.23	2989.05	2700.39
51	9868.12	9860.34	9850.59	9838.38	100	3102.90	2895.94	2652.63	2371.88
52	9855.48	9847.01	9836.39	9823.08	101	2763.19	2561.21	2325.37	2055.64
53	9841.72	9832.48	9820.90	9806.39	102	2431.39	2236.61	2010.90	1755.27
54	9826.71	9816.64	9804.02	9788.18	103	2111.15	1925.80	1712.81	1474.18
55	9810.34	9799.37	9785.60	9768.33	104	1806.12	1632.34	1434.48	1215.44
56	9792.49	9780.52	9765.51	9746.67	105	1519.82	1359.55	1178.94	981.65
57	9773.03	9759.97	9743.60	9723.05	106	1255.46	1110.36	948.70	774.71
58	9751.79	9737.56	9719.69	9697.28	107	1015.81	887.14	745.58	595.71
59	9728.63	9713.10	9693.62	9669.17	108	802.96	691.49	570.56	444.87
60	9703.36	9686.43	9665.17	9638.51	109	618.23	524.17	423.71	321.41
61	9675.80	9657.33	9634.15	9605.07	110	462.04	385.00	304.13	223.65
62	9645.73	9625.59	9600.31	9568.61	111	333.80	272.80	210.00	149.10
63	9612.94	9590.98	9563.42	9528.85	112	231.99	185.53	138.71	94.62
64	9577.18	9553.24	9523.19	9485.52	113	154.19	120.34	87.07	56.74
65	9538.19	9512.09	9479.35	9438.30	114	97.30	73.90	51.50	31.84
66	9495.69	9467.25	9431.58	9386.86	115	57.78	42.55	28.41	16.52
67	9449.37	9418.39	9379.54	9330.85	116	31.92	22.69	14.43	7.81
68	9398.90	9365.17	9322.87	9269.88	117	16.15	11.04	6.63	3.30
69	9343.95	9307.23	9261.20	9203.55	118	7.34	4.79	2.69	1.21
70	9284.12	9244.18	9194.11	9131.43	119	2.90	1.79	0.93	0.37
71	9219.03	9175.59	9121.17	9053.07	120	0.95	0.55	0.26	0.09
72	9148.24	9101.03	9041.91	8967.97	121	0.23	0.13	0.05	0.01
73	9071.30	9020.03	8955.85	8875.63	122	0.03	0.02	0.01	0.00