ACSC/STAT 3720, Life Contingencies I
Winter 2015
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Homework Sheet 3
Model Solutions

Basic Questions

1. *Calculate the expected benefit of a whole life insurance sold to an individual aged 96, if the death benefit is \$1,200,000 at the end of the year of death, the lifetable is Table 1, and the interest rate is $i = 0.06$.*

$$\begin{array}{lll} A_{125} = 1 & A_{124} = 0.943396 & A_{123} = 0.937463 \\ A_{122} = 0.929046 & A_{121} = 0.922928 & A_{120} = 0.916736 \\ A_{119} = 0.910206 & A_{118} = 0.903348 & A_{117} = 0.896059 \\ A_{116} = 0.88837 & A_{115} = 0.880256 & A_{114} = 0.8717 \\ A_{113} = 0.862704 & A_{112} = 0.853268 & A_{111} = 0.843386 \\ A_{110} = 0.833061 & A_{109} = 0.822293 & A_{108} = 0.81109 \\ A_{107} = 0.799459 & A_{106} = 0.787411 & A_{105} = 0.774958 \\ A_{104} = 0.762114 & A_{103} = 0.748897 & A_{102} = 0.735325 \\ A_{101} = 0.72142 & A_{100} = 0.707204 & A_{99} = 0.692702 \\ A_{98} = 0.67794 & A_{97} = 0.662946 & A_{96} = 0.647747 \end{array}$$

   The expected death benefit is therefore $1200000 \times 0.647747 = \$777,296.40$.

2. *Calculate the expected benefit, and the variance of the benefit of a 5-year term policy with benefit \$200,000 at the end of year of death of the policyholder. The lifetable for this policy is Table 1, and the interest rate is $i = 0.04$. The policy is sold to an individual aged 41.*

   From the lifetable, we calculate

$$\begin{aligned} A^1_{46:\overline{0}|} &= 0 \\ A^1_{45:\overline{1}|} &= 0.000689415 \\ A^1_{44:\overline{2}|} &= 0.00129723 \\ A^1_{43:\overline{3}|} &= 0.00183068 \\ A^1_{42:\overline{4}|} &= 0.00229763 \\ A^1_{41:\overline{5}|} &= 0.00270465 \end{aligned}$$

   To calculate the expected square of the present value, we calculate at interest rate $i = (1.04)^2 - 1 = 0.0816$, using the same recurrence.

$$
\begin{aligned}
{}^2A^1_{46:\overline{0}|} &= 0 \\
{}^2A^1_{45:\overline{1}|} &= 0.000662899 \\
{}^2A^1_{44:\overline{2}|} &= 0.00122284 \\
{}^2A^1_{43:\overline{3}|} &= 0.00169153 \\
{}^2A^1_{42:\overline{4}|} &= 0.00208068 \\
{}^2A^1_{41:\overline{5}|} &= 0.00240015
\end{aligned}
$$

The variance of the present value of a payment of \$1 at end of the year of death, provided it happens within 5 years is therefore ${}^2A^1_{41:\overline{5}|} - (A^1_{41:\overline{5}|})^2 = 0.00240015 - 0.00270465^2 = 0.002392834868$. The variance of the present value of future payments is therefore $0.002392834868 \times 200000^2 = 95713394.72$.

3. *A select individual aged 42 purchases a 5-year endowment insurance policy with a benefit of \$100,000 either at the end of 5 years or payable immediately upon the death of the individual. Force of interest is $\delta = 0.038$. Using a uniform distribution of deaths assumption, calculate the expected benefit from this policy.*

   Using our standard recurrence, we get $i = e^{0.038} - 1 = 0.03873123288$. This gives

$$
\begin{aligned}
A^1_{47:\overline{0}|} &= 0 \\
A^1_{46:\overline{1}|} &= 0.00075090 \\
A^1_{45:\overline{2}|} &= 0.00141264 \\
A^1_{[42]+2:\overline{3}|} &= 0.001911463 \\
A^1_{[42]+1:\overline{4}|} &= 0.002259654 \\
A^1_{[42]:\overline{5}|} &= 0.002528923
\end{aligned}
$$

   Under UDD, we get $\overline{A}^1_{[42]:\overline{5}|} = \frac{i}{\delta} A^1_{[42]:\overline{5}} = \frac{0.03873123288 \times 0.002528923}{0.038} = 0.002577587$. We then get $\overline{A}_{[42]:\overline{5}|} = \overline{A}^1_{[42]:\overline{5}|} +_5 p_{[42]} e^{-0.19} = 0.002577587 + \frac{9915.52}{9944.32} e^{-0.19} = 0.8271417$.

   The expected benefit is therefore $0.8271417 \times 100000 = \$82,714.17$.

4. *An individual aged 39 wants to purchase whole life insurance that pays a benefit at the end of the year of death. The interest rate is $i = 0.06$. The individual has a number of dangerous hobbies and uses the special lifetable:*

x	l_x	d_x
39	*10000.00*	*4.80*
40	*9995.20*	*4.86*
41	*9990.34*	*4.93*
42	*9985.41*	*5.01*
43	*9980.40*	*5.09*
44	*9975.31*	*5.18*
45	*9970.13*	*5.29*

After age 45, the individual will be too old to participate in these hobbies and will use a standard lifetable, which will give the value $A_{45} = 0.1761$. Calculate the EPV of the benefit for this individual from a whole-life policy which has a death benefit of \$200,000.

Using the standard recurrence, we calculate:

$$\begin{aligned} A_{45} &= 0.1761 \\ A_{44} &= 0.166536 \\ A_{43} &= 0.15751 \\ A_{42} &= 0.148993 \\ A_{41} &= 0.140956 \\ A_{40} &= 0.133371 \\ A_{39} &= 0.126214 \end{aligned}$$

This means the EPV of the benefit is $200000 \times 0.126214 = 25,242.80$.

Standard Questions

5. *A select individual aged 42 has a 15-year endowment insurance with a benefit of \$800,000 payable either at the end of the year of death or at the end of 15 years. The individual wants to convert this to a whole life insurance policy. If the current interest rate is $i = 0.06$, what benefit for the whole life policy would have the same EPV as the endowment insurance policy? [The company has already calculated that $A_{57} = 0.150748$ and $A_{[42]} = 0.07106117$.]*

   The EPV of the endowment insurance is

$$\begin{aligned} 800000 A_{[42]:\overline{15}|} &= 800000(A_{[42]} +_{15} p_{[42]}(1.06)^{-15}(1 - A_{57})) \\ &= 800000(0.07106117 + \frac{9788.18}{9944.32}(1.06)^{-15}(1 - 0.150748)) \\ &= \$335,888.28 \end{aligned}$$

   To get a whole-life insurance policy with the same EPV of benefits, we calculate $\frac{335888.28}{0.07106117} = \$4,726,748.52$.

6. *A woman aged 35 buys a house with a mortgage of \$400,000. She amortises this amount with annual payments over a period of 25 years at $i = 0.07$. She takes out mortgage insurance, which pays off the outstanding balance (principle plus interest) of the mortgage at the end of the year in which she dies. [Assume that the mortgage company does not charge a penalty for early repayment in this case.] If the insurance company uses an interest rate $i = 0.05$ and the ultimate part of the life table from Table 1, calculate the expected present value of the benefit on this policy. You are given the following values, some of which may be useful:*

i	$A_{35:\overline{26}\vert}$
-0.02	*1.68411*
-0.01904762	*1.64239*
-0.01869159	*1.62708*
0.01869159	*0.620828*
0.01904762	*0.615276*
0.02	*0.600679*
0.05	*0.285768*
0.07	*0.176736*

First we calculate the mortgage payments. We calculate $a_{\overline{25}|0.07} = \frac{1-(1.07)^{-25}}{0.07} = 11.65358$, so the mortgage payments are $\frac{400000}{11.65358} = \$34,324.21$. The accumulated value of the first $n-1$ years of mortgage payments after n years is therefore $34324.21\frac{(1.07^n-1.07)}{0.07}$, while the interest on the original 400,000 has increased it to $400000(1.07)^n$, so the outstanding balance just before the nth payment is

$$400000(1.07)^n - \frac{34324.21}{0.07}(1.07)^n + \frac{34324.21 \times 1.07}{0.07} = 524670 - 90345.81(1.07)^n$$

If the payments are increasing at 0.07, the "real rate of interest is $\frac{0.05-0.07}{1.07} = -0.01869159$ The EPV of the payment is therefore $524670A_{35:\overline{26}|} - 90345.81^{-0.01869159}A_{35:\overline{26}|} = 524670 \times 0.285768 - 90345.81 \times 1.62708 = \$2934.04$.

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66
26	9997.00	9995.83	9994.40	9992.66
27	9995.14	9993.90	9992.38	9990.52
28	9993.16	9991.84	9990.22	9988.24
29	9991.05	9989.65	9987.92	9985.80
30	9988.81	9987.30	9985.46	9983.18
31	9986.40	9984.80	9982.82	9980.38
32	9983.83	9982.11	9979.99	9977.37
33	9981.07	9979.23	9976.95	9974.13
34	9978.11	9976.13	9973.68	9970.64
35	9974.93	9972.79	9970.16	9966.88
36	9971.50	9969.20	9966.36	9962.82
37	9967.80	9965.33	9962.25	9958.44
38	9963.81	9961.14	9957.82	9953.69
39	9959.50	9956.61	9953.02	9948.55
40	9954.84	9951.71	9947.82	9942.98
41	9949.79	9946.41	9942.19	9936.94
42	9944.32	9940.66	9936.08	9930.38
43	9938.39	9934.41	9929.45	9923.26
44	9931.96	9927.64	9922.25	9915.52
45	9924.97	9920.28	9914.42	9907.10
46	9917.37	9912.28	9905.91	9897.94
47	9909.11	9903.58	9896.65	9887.98
48	9900.13	9894.11	9886.57	9877.13
49	9890.36	9883.80	9875.59	9865.30
50	9879.71	9872.57	9863.63	9852.42
51	9868.12	9860.34	9850.59	9838.38
52	9855.48	9847.01	9836.39	9823.08
53	9841.72	9832.48	9820.90	9806.39
54	9826.71	9816.64	9804.02	9788.18
55	9810.34	9799.37	9785.60	9768.33
56	9792.49	9780.52	9765.51	9746.67
57	9773.03	9759.97	9743.60	9723.05
58	9751.79	9737.56	9719.69	9697.28
59	9728.63	9713.10	9693.62	9669.17
60	9703.36	9686.43	9665.17	9638.51
61	9675.80	9657.33	9634.15	9605.07
62	9645.73	9625.59	9600.31	9568.61
63	9612.94	9590.98	9563.42	9528.85
64	9577.18	9553.24	9523.19	9485.52
65	9538.19	9512.09	9479.35	9438.30
66	9495.69	9467.25	9431.58	9386.86
67	9449.37	9418.39	9379.54	9330.85
68	9398.90	9365.17	9322.87	9269.88
69	9343.95	9307.23	9261.20	9203.55
70	9284.12	9244.18	9194.11	9131.43
71	9219.03	9175.59	9121.17	9053.07
72	9148.24	9101.03	9041.91	8967.97
73	9071.30	9020.03	8955.85	8875.63
74	8987.73	8932.10	8862.49	8775.52
75	8897.04	8836.71	8761.27	8667.10
76	8798.69	8733.34	8651.66	8549.78
77	8692.13	8621.41	8533.09	8423.00
78	8576.81	8500.36	8404.95	8286.16
79	8452.13	8369.60	8266.68	8138.66
80	8317.52	8228.53	8117.67	7979.93
81	8172.36	8076.57	7957.35	7809.41
82	8016.08	7913.13	7785.15	7626.56
83	7848.11	7737.67	7600.54	7430.89
84	7667.89	7549.66	7403.05	7221.99
85	7474.92	7348.64	7192.27	6999.51
86	7268.77	7134.21	6967.86	6763.22
87	7049.07	6906.07	6729.62	6513.04
88	6815.55	6664.05	6477.46	6249.02
89	6568.09	6408.10	6211.48	5971.42
90	6306.70	6138.35	5931.96	5680.73
91	6031.59	5855.15	5639.41	5377.67
92	5743.19	5559.08	5334.61	5063.27
93	5442.15	5250.97	5018.61	4738.86
94	5129.44	4931.97	4692.79	4406.12
95	4806.33	4603.54	4358.89	4067.08
96	4474.39	4267.51	4018.96	3724.10
97	4135.60	3926.04	3675.44	3379.91
98	3792.25	3581.66	3331.11	3037.57
99	3447.02	3237.23	2989.05	2700.39
100	3102.90	2895.94	2652.63	2371.88
101	2763.19	2561.21	2325.37	2055.64
102	2431.39	2236.61	2010.90	1755.27
103	2111.15	1925.80	1712.81	1474.18
104	1806.12	1632.34	1434.48	1215.44
105	1519.82	1359.55	1178.94	981.65
106	1255.46	1110.36	948.70	774.71
107	1015.81	887.14	745.58	595.71
108	802.96	691.49	570.56	444.87
109	618.23	524.17	423.71	321.41
110	462.04	385.00	304.13	223.65
111	333.80	272.80	210.00	149.10
112	231.99	185.53	138.71	94.62
113	154.19	120.34	87.07	56.74
114	97.30	73.90	51.50	31.84
115	57.78	42.55	28.41	16.52
116	31.92	22.69	14.43	7.81
117	16.15	11.04	6.63	3.30
118	7.34	4.79	2.69	1.21
119	2.90	1.79	0.93	0.37
120	0.95	0.55	0.26	0.09
121	0.23	0.13	0.05	0.01
122	0.03	0.02	0.01	0.00