ACSC/STAT 3720, Life Contingencies I
Winter 2016
Toby Kenney
Homework Sheet 6
Model Solutions

Basic Questions

1. *Using the lifetable in Table 1, and interest rate $i = 0.05$, calculate the net annual premium for a 5-year endowment insurance policy with benefit \$350,000, sold to a standard life aged 44, if:*

   (a) The life works in a hazardous environment, and has mortality 0.008 higher than normal.

   For this higher than normal mortality, we have that $\ddot{a}_{44:\overline{5}|}$ is the value of $\ddot{a}_{44:\overline{5}|}$ calculated at a force of interest which is 0.008 higher than the true force of interest. That is at $i = e^{(\log(1.05)+0.008)} - 1 = 1.05e^{0.008} - 1 = 0.05843369$. Using the standard recurrence, we get $A_{44:\overline{5}|} = 0.753138$, so $\ddot{a}_{44:\overline{5}|} = \frac{1.05843369(1-0.753138)}{0.05843369} = 4.471513908$. Now to find the value of $A_{44:\overline{5}|}$, for this life, we calculate it from $\ddot{a}_{44:\overline{5}|}$ at the actual interest rate, to get

   $$A_{44:\overline{5}|} = 1 - \frac{0.05}{1.05}\ddot{a}_{44:\overline{5}|} = 0.7870707663$$

   The premium is therefore $\frac{0.7870707663\times 350000}{4.471513908} = \$61,606.60$.

   (b) The life is an impaired life, and has mortality 1.06 times the usual mortality for a life of the same age.

   For this life we get that $p'_x = (p_x)^{1.06}$, where p'_x is the probability of survival for this life and p_x is the probability of survival for a standard life. From this, we calculate

   $$\ddot{a}_{44:\overline{5}|} = 1+\left(\frac{9930.38}{9936.94}\right)^{1.06}(1.05)^{-1}+\left(\frac{9923.26}{9936.94}\right)^{1.06}(1.05)^{-2}+\left(\frac{9915.52}{9936.94}\right)^{1.06}(1.05)^{-3}+\left(\frac{9907.10}{9936.94}\right)^{1.06}(1.05)^{-4} =$$

   This gives

   $$A_{44:\overline{5}|} = 1 - \frac{0.05}{1.05}\times 4.539368312 = 0.7838396042$$

   The premium is therefore $\frac{350000\times 0.7838396042}{4.539368312} = \$60,436.57$.

2. *An insurance company has a whole life insurance policy for an individual aged 52. The death benefit of this policy is \$800,000, and the interest rate is $i = 0.06$. Premiums are payable until age 80. The insurance company calculates $A_{52} = 0.118287$, and $A_{80} = 0.400802$. Therefore, the net annual premium for the policy is \$6,852.85. What is the policy value if the life survives to age 65? [Use the lifetable in Table 1. $A_{65} = 0.218135$.]*

   We calculate $\ddot{a}_{65} = \frac{1.06(1-0.218135)}{0.06} = 13.81295$, and $\ddot{a}_{80} = \frac{1.06(1-0.400802)}{0.06} = 10.58583$. This gives $\ddot{a}_{65:\overline{15}|} = 13.81295 - \frac{8423.00}{9568.61}10.58583(1.06)^{-28} = 9.924694$. This allows us to calculate

   $${}_{13}V = 800000\times 0.218135 - 6852.85\times 9.924694 = \$106,495.56$$

3. *An insurance company sells 600 whole life insurance policies with annual net premiums to lives aged 53. The death benefit on these policies is \$400,000. The interest rate is $i = 0.06$. In the first year of the policies:*

- *One policyholder dies.*
- *The company earns interest $i = 0.07$.*

The company still uses $i = 0.06$ as its basis for calculating the policy values. What is the company's annual profit on these policies? [Using the lifetable in Table 1, we have $A_{53} = 0.124241$ and $A_{54} = 0.130456$.]

We calculate $\ddot{a}_{53} = \frac{1.06(1-0.124241)}{0.06} = 15.47174$, so the premium is $400000\frac{0.124241}{15.47174} = \$3,212.08$. After 1 year, we have $\ddot{a}_{54} = \frac{1.06(1-0.130456)}{0.06} = 15.36194$, so the policy value is $0.130456 \times 400000 - 15.36194 \times 3212.08 = 2838.68$.

At the start of the first year, the company receives $600 \times 3212.08 = \$1,927,246$ in premiums, after earning 7% interest on this, it has $1927246 \times 1.07 = \$2,062,152.79$. At the end of the year, it pays one death benefit of $\$400,000$ and it has 599 policies in force, each with value $\$2838.68$, so the costs are $2062152.79 - 400000 - 599 \times 2838.68 = \$-38,217.13$.

4. *An insurance company sells 300 whole-life insurance policies to lives aged 45. The death benefit of these policies is \$800,000. The interest rate is $i = 0.045$ and net premiums are payable annually in advance. At this interest rate, $A_{45} = 0.142031$. In the first two years of the policy:*

   - *two policyholders die in the first year of the policy.*
   - *The company earns interest $i = 0.06$ in the first year of the policy, and $i = 0.05$ in the second year.*

   Calculate the asset share of the remaining policies after the second year.

   We calculate $\ddot{a}_{45} = \frac{1.045(1-0.142031)}{0.045} = 19.92394678$. This gives the premium as $\frac{800000\times 0.142031}{19.92394678} = \$5,702.93$. At the start of the first year, the company receives $5702.93 \times 300 = 1710877.89$ in premiums. After earning interest, at the end of the year it has $1710877.889 \times 1.06 = \$1,813,530.56$. From this, it pays out two death benefits of \$800,000, so it has $\$213,530.56$. It then receives $5702.93\times 298 = \$1,699,472.04$ in premiums for year 2, so it has \$1,913,002.60. At the end of the year, with the interest it has $1913002.598 \times 1.05 = \$2,008,652.73$. It does not pay any death benefits in that year, so the asset share at the end of the year is $\frac{2008652.73}{298} = \$6,740.45$.

5. *A select life aged 37 purchases a whole-life insurance policy with a death benefit of \$600,000. The interest rate is $i = 0.05$. From the lifetable in Table 1, we have $A_{37} = 0.0827855$ and $A_{43} = 0.108129$. Using Woolhouse's formula:*

   (a) calculate the monthly premium.

   We calculate $\ddot{a}_{37} = \frac{1.05(1-0.0827855)}{0.05} = 19.2615045$, and we approximate

$$\mu_{37} = \frac{1}{2}(q_{36} + q_{37}) = \frac{1}{2}\left(\frac{3.49}{9974.13} + \frac{3.76}{9970.64}\right) = 0.0003635061957$$

   and calculate $\delta = \log(1.05) = 0.04879016417$ so using Woolhouse's formula:

$$\ddot{a}_{37}^{(12)} = 19.2615045 - \frac{11}{24} - \frac{143}{1728}(0.04879016417 + 0.0003635061957) = 18.79910347$$

   We then calculate $d^{(12)} = 12(1 - 1.05^{-\frac{1}{12}} = 0.04869111179$, and $A_{37}^{(12)} = 1 - 0.04869111179 \times 18.79910347 = 0.08465075139$, so the monthly premium is $600000 \times 0.0846507513912 \times 18.79910347 = \$225.15$.

(b) calculate the policy value after 6 years and 4 months. [You may use the UDD assumption for the distribution of deaths in Year 7, but use Woolhouse's formula to calculate $\ddot{a}_{43}^{(12)}$.]

We calculate $\ddot{a}_{43} = \frac{1.05(1-0.108129)}{0.05} = 18.72929$. We approximate

$$\mu_{43} = \frac{1}{2}(q_{42} + q_{43}) = \frac{1}{2}\left(\frac{6.04}{9942.98} + \frac{6.56}{9936.94}\right) = 0.0006338134$$

using Woolhouse's formula:

$$\ddot{a}_{43}^{(12)} = 18.72929 - \frac{11}{24} - \frac{143}{1728}(0.04879016417 + 0.0006338134) = 18.2668676$$

Now we have that $q_{43} = \frac{6.56}{9936.94} = 0.0006601629878$, so

$$a_{43}^{(12)} = \frac{1}{12}\left(1 + \left(1 - \frac{q_{43}}{12}\right)(1.05)^{-\frac{1}{12}} + \left(1 - \frac{2q_{43}}{12}\right)(1.05)^{-\frac{1}{12}} + \left(1 - \frac{3q_{43}}{12}\right)(1.05)^{-\frac{3}{12}}\right) + \left(1 - \frac{4q_{43}}{12}\right)(1.05)^{-\frac{4}{12}}\ddot{a}_{43\frac{4}{12}}^{(12)}$$

We solve this to get

$$\ddot{a}_{43\frac{4}{12}}^{(12)} = \frac{18.2668676 - 0.3312827723}{0.9836516424} = 18.23367547$$

This gives

$$A_{43\frac{4}{12}}^{(12)} = 1 - d^{(12)}\ddot{a}_{43\frac{4}{12}}^{12} = 1 - 0.04869111179 \times 18.23367547 = 0.1121820693$$

The policy value is therefore $600000 \times 0.1121820693 - 225.15 \times 12 \times 18.23367547 = \$18,046.44$.

(c) calculate the policy value after 6 years 3.6 months.

After 6 years 3.6 months, the policy pays \$600000 at the end of the month if the life dies before the end of the month, otherwise the policy has value \$18,046.44 at the end of the month. The probability of dying before the end of the month is $\frac{\frac{0.4}{12}q_{43}}{1-\frac{3.6}{12}q_{43}} = 0.00002200979194$, so the policy value is

$$(0.00002200979194 \times 600000 + 0.99997799021 \times 18046.44)(1.05)^{-\frac{0.4}{12}} = \$18,029.90$$

Standard Questions

6. *A select life aged 39 takes out a whole life insurance with benefit \$600,000. The initial cost of this insurance is \$1000 plus 30% of the first annual premium. The renewal cost is 2% of each subsequent premium. The interest rate is $i = 0.05$. Using the lifetable in Table 1, we can calculate $A_{42} = 0.103456$.*

   (a) Calculate the gross premium for this policy.

   Using the standard recurrence, we calculate

$$
\begin{aligned}
A_{[39]+2} &= 0.09891299695 \\
A_{[39]+1} &= 0.09451228279 \\
A_{[39]} &= 0.09026193607
\end{aligned}
$$

This gives $\ddot{a}_{[39]} = \frac{1.05(1-0.09026193607)}{0.05} = 19.10449934$. The EPV of benefits is therefore $600000 \times 0.09026193607 = \$54,157.16$, while the cost of premiums less expenses is $(19.10449934 \times 0.98 - 0.28)P - 1000 = 18.44240936P - 1000$. We therefore need to solve

$$
\begin{aligned}
18.44240936P - 1000 &= 54157.16 \\
P &= \frac{55157.16}{18.44240936} = \$2,990.78
\end{aligned}
$$

(b) Calculate the gross policy value after 2 years.

After 2 years, we have $A_{[39]+2} = 0.09891299695$, so $\ddot{a}_{[39]+2} = \frac{1.05(1-0.09891299695)}{0.05} = 18.9228270618.9424230419.157$ so the policy value is

$$
{}_2V = 600000 \times 0.09891299695 - 2990.78 \times 0.98 \times 18.92282706 = \$3,885.69
$$

7. *An insurance company wants to design a 10-year term policy with continuous premiums so that the policy value is given by ${}_tV = 150t(t-10)(t-15)$. The death benefits at time t are $100000(3+t)$. The policy is sold to a life aged 52, with mortality given by $\mu_x = 0.0000012(1.106)^x$. Calculate the premiums as a function of time if force of interest is $\delta = 0.051$.*

   Thiel's differential equation gives

$$
\frac{d}{dt}\,{}_tV = \delta_t\,{}_tV + P_t - (S_t - {}_tV)\mu_{x+t}
$$

   We substitute the given values to get

$$
\begin{aligned}
150(3t^2 - 50t + 150) &= 0.051 \times 150t(t-10)(t-15) + P_t - (100000(3+t) - 150t(t-10)(t-15))0.0000012(1.106)^{52+t} \\
P_t &= 150(3t^2 - 50t + 150) - 7.65t(t-10)(t-15) + (100000(3+t) - 150t(t-10)(t-15))0.0000012(1.106)^{52+t}
\end{aligned}
$$

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$	x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66	74	8987.73	8932.10	8862.49	8775.52
26	9997.00	9995.83	9994.40	9992.66	75	8897.04	8836.71	8761.27	8667.10
27	9995.14	9993.90	9992.38	9990.52	76	8798.69	8733.34	8651.66	8549.78
28	9993.16	9991.84	9990.22	9988.24	77	8692.13	8621.41	8533.09	8423.00
29	9991.05	9989.65	9987.92	9985.80	78	8576.81	8500.36	8404.95	8286.16
30	9988.81	9987.30	9985.46	9983.18	79	8452.13	8369.60	8266.68	8138.66
31	9986.40	9984.80	9982.82	9980.38	80	8317.52	8228.53	8117.67	7979.93
32	9983.83	9982.11	9979.99	9977.37	81	8172.36	8076.57	7957.35	7809.41
33	9981.07	9979.23	9976.95	9974.13	82	8016.08	7913.13	7785.15	7626.56
34	9978.11	9976.13	9973.68	9970.64	83	7848.11	7737.67	7600.54	7430.89
35	9974.93	9972.79	9970.16	9966.88	84	7667.89	7549.66	7403.05	7221.99
36	9971.50	9969.20	9966.36	9962.82	85	7474.92	7348.64	7192.27	6999.51
37	9967.80	9965.33	9962.25	9958.44	86	7268.77	7134.21	6967.86	6763.22
38	9963.81	9961.14	9957.82	9953.69	87	7049.07	6906.07	6729.62	6513.04
39	9959.50	9956.61	9953.02	9948.55	88	6815.55	6664.05	6477.46	6249.02
40	9954.84	9951.71	9947.82	9942.98	89	6568.09	6408.10	6211.48	5971.42
41	9949.79	9946.41	9942.19	9936.94	90	6306.70	6138.35	5931.96	5680.73
42	9944.32	9940.66	9936.08	9930.38	91	6031.59	5855.15	5639.41	5377.67
43	9938.39	9934.41	9929.45	9923.26	92	5743.19	5559.08	5334.61	5063.27
44	9931.96	9927.64	9922.25	9915.52	93	5442.15	5250.97	5018.61	4738.86
45	9924.97	9920.28	9914.42	9907.10	94	5129.44	4931.97	4692.79	4406.12
46	9917.37	9912.28	9905.91	9897.94	95	4806.33	4603.54	4358.89	4067.08
47	9909.11	9903.58	9896.65	9887.98	96	4474.39	4267.51	4018.96	3724.10
48	9900.13	9894.11	9886.57	9877.13	97	4135.60	3926.04	3675.44	3379.91
49	9890.36	9883.80	9875.59	9865.30	98	3792.25	3581.66	3331.11	3037.57
50	9879.71	9872.57	9863.63	9852.42	99	3447.02	3237.23	2989.05	2700.39
51	9868.12	9860.34	9850.59	9838.38	100	3102.90	2895.94	2652.63	2371.88
52	9855.48	9847.01	9836.39	9823.08	101	2763.19	2561.21	2325.37	2055.64
53	9841.72	9832.48	9820.90	9806.39	102	2431.39	2236.61	2010.90	1755.27
54	9826.71	9816.64	9804.02	9788.18	103	2111.15	1925.80	1712.81	1474.18
55	9810.34	9799.37	9785.60	9768.33	104	1806.12	1632.34	1434.48	1215.44
56	9792.49	9780.52	9765.51	9746.67	105	1519.82	1359.55	1178.94	981.65
57	9773.03	9759.97	9743.60	9723.05	106	1255.46	1110.36	948.70	774.71
58	9751.79	9737.56	9719.69	9697.28	107	1015.81	887.14	745.58	595.71
59	9728.63	9713.10	9693.62	9669.17	108	802.96	691.49	570.56	444.87
60	9703.36	9686.43	9665.17	9638.51	109	618.23	524.17	423.71	321.41
61	9675.80	9657.33	9634.15	9605.07	110	462.04	385.00	304.13	223.65
62	9645.73	9625.59	9600.31	9568.61	111	333.80	272.80	210.00	149.10
63	9612.94	9590.98	9563.42	9528.85	112	231.99	185.53	138.71	94.62
64	9577.18	9553.24	9523.19	9485.52	113	154.19	120.34	87.07	56.74
65	9538.19	9512.09	9479.35	9438.30	114	97.30	73.90	51.50	31.84
66	9495.69	9467.25	9431.58	9386.86	115	57.78	42.55	28.41	16.52
67	9449.37	9418.39	9379.54	9330.85	116	31.92	22.69	14.43	7.81
68	9398.90	9365.17	9322.87	9269.88	117	16.15	11.04	6.63	3.30
69	9343.95	9307.23	9261.20	9203.55	118	7.34	4.79	2.69	1.21
70	9284.12	9244.18	9194.11	9131.43	119	2.90	1.79	0.93	0.37
71	9219.03	9175.59	9121.17	9053.07	120	0.95	0.55	0.26	0.09
72	9148.24	9101.03	9041.91	8967.97	121	0.23	0.13	0.05	0.01
73	9071.30	9020.03	8955.85	8875.63	122	0.03	0.02	0.01	0.00