ACSC/STAT 3720, Life Contingencies I
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Homework Sheet 8
Model Solutions

Basic Questions

1. *A woman aged 32, for whom the ultimate part of the lifetable in Table 1 is appropriate, buys a 20-year term insurance policy with a death benefit of \$800,000. (The policy uses a net annual premium.) Five years later, she wants to surrender the policy. The interest rate is $i = 0.04$. This gives $A_{32} = 0.106825$ $A_{37} = 0.128530$ and $A_{52} = 0.220514$. If the insurance company pays a cash surrender value of 80% of the net policy value, how much does she receive?*

   We calculate

$$\begin{aligned}
\ddot{a}_{32} &= \frac{1.04}{0.04}(1-0.106825) = 23.22255\\
\ddot{a}_{37} &= \frac{1.04}{0.04}(1-0.128530) = 22.65822\\
\ddot{a}_{52} &= \frac{1.04}{0.04}(1-0.220514) = 20.26664\\
A^1_{32:\overline{20}} &= 0.106825 - \frac{9865.30}{9985.80}(1.04)^{-20}\times 0.220514 = 0.007399722\\
A^1_{37:\overline{15}} &= 0.128530 - \frac{9865.30}{9970.64}(1.04)^{-15}\times 0.220514 = 0.007380022\\
\ddot{a}_{32:\overline{20}} &= 23.22255 - \frac{9865.30}{9985.80}(1.04)^{-20}\times 20.26664 = 14.08473\\
\ddot{a}_{32:\overline{20}} &= 22.65822 - \frac{9865.30}{9985.80}(1.04)^{-15}\times 20.26664 = 11.52377
\end{aligned}$$

   This means that the premium for the policy is $\frac{800000\times 0.007399722}{14.08473} = \$420.30$. The policy value after 5 years is therefore $800000\times 0.007380022 - 420.30\times 11.52377 = \$1,060.58$. The surrender value is $1060.58\times 0.8 = \$848.46$.

2. *An insurance company sells a 15-year term insurance policy to a life aged 29 to whom the ultimate part of the lifetable in Table 1 applies. The death benefit is \$180,000 in the first two years, \$160,000 in the third to fifth year and \$140,000 for the remaining 10 years. The premiums are \$96.85 for the first three years, and \$26.64 for the remaining twelve years. The interest rate is $i = 0.05$ for the first 4 years, and $i = 0.07$ for the remaining 11 years. Calculate the retrospective policy value after 1 year.*

   In the first year, we have $p_{29} = \frac{9990.52}{9992.66}$ and $q_{29} = \frac{2.14}{9992.66}$, so the expected death benefits paid at the end of the first year are $180000\times\frac{2.14}{9992.66} = 38.54829$. we therefore get the equation:

$$
\begin{aligned}
96.85 &= \left(38.54829 + \frac{9990.52}{9992.66}{}_1V\right)(1.05)^{-1} \\
101.6925 &= 38.54829 + \frac{9990.52}{9992.66}{}_1V \\
\frac{9990.52}{9992.66}{}_1V &= 63.14421 \\
{}_1V &= \$63.16
\end{aligned}
$$

3. *A man aged 47, who is a select life on Table 1 buys a whole life insurance with a benefit of \$500,000. The interest rate is $i = 0.05$, which gives $A_{[47]} = 0.1283161$, $A_{[47]+2} = 0.1403596$, $A_{52} = 0.159677$ and $A_{54} = 0.173724$. Using a Full preliminary term of 2 years, calculate the policy value after 5 years.*

   When we use a full preliminary term, we calculate the policy value based on the net premium that would apply if the policy were purchased at the end of the preliminary term. In this case, at age $[47]+2$, so for this policy we have $\ddot{a}_{[47]+2} = \frac{1.05}{0.05}(1 - 0.1403596) = 18.05245$ and the premium would be $\frac{500000 \times 0.1403596}{18.05245} = \$3,887.55$. Using this premium, at age 52, we have $\ddot{a}_{52} = \frac{1.05}{0.05}(1 - 0.159677) = 17.64678$ so the policy value is $500000 \times 0.159677 - 3887.55 \times 17.64678 = \$11,235.76$.

Standard Questions

4. *A man aged 38, who is a select life on Table 1 buys a 10-year annual endowment insurance policy with a benefit of \$400,000. The interest rate is $i = 0.06$, so $A_{[38]:\overline{10}|} = 0.5591854$ $A_{43:\overline{5}|} = 0.747573$ and $A_{[43]:\overline{5}|} = 0.7475003$. The insurance company pays a cash surrender value of 85% of the policy value. If he is still a select life at age 43, would he save money by surrendering his current policy and buying a new 5-year policy for the same coverage?*

   We have $\ddot{a}_{[38]:\overline{10}|} = \frac{1.06}{0.06}(1 - 0.5591854) = 7.787725$, so the premium is $\frac{400000 \times 0.5591854}{7.787725} = \$28,721.37$. We have $\ddot{a}_{43:\overline{5}|} = \frac{1.06}{0.06}(1 - 0.747573) = 4.459544$, so the policy value after 5 years is $400000 \times 0.747573 - 28721.37 \times 4.459544 = \$170,944.99$. The cash surrender value is $170944.99 \times 0.85 = \$145,303.24$. For a new 5-year policy, the EPV of benefits is $400000 \times 0.7475003 = \$299,000.12$. Subtracting the cash surrender value gives $299000.12 - 145303.24 = \$153,696.88$. The new premium is therefore $\frac{153,696.88}{4.460828} = \$34,454.79$, so he would not save money by surrendering and repurchasing.

5. *A man bought a whole life insurance policy 6 years ago. At the time, his age was 42, and his mortality followed the ultimate part of the lifetable in Table 1. The interest rate is $i = 0.06$. He now wants to convert the policy to a paid-up term policy with the same death benefit. The insurance company offers a cash surrender value of 80% of the policy value. What is the term of the new insurance contract? You are given the following values of A_x:*

x	A_x	x	A_x	x	A_x
42	*0.0714153*	*55*	*0.136941*	*63*	*0.199371*
48	*0.0969315*	*56*	*0.143702*	*64*	*0.208588*
49	*0.101917*	*57*	*0.150748*	*65*	*0.218135*
50	*0.107134*	*58*	*0.158086*	*66*	*0.228016*
51	*0.112588*	*59*	*0.165721*	*67*	*0.238233*
52	*0.118287*	*60*	*0.173662*	*68*	*0.248787*
53	*0.124241*	*61*	*0.181913*	*69*	*0.259679*
54	*0.130456*	*62*	*0.190481*	*70*	*0.27091*

We have $\ddot{a}_{42} = \frac{1.06}{0.06}(1 - A_{42}) = 16.40500$ so if without loss of generality, we let the benefit be 1, then the premium is $\frac{0.0714153}{16.40500} = 0.004353264$. After 6 years, we have $\ddot{a}_{48} = \frac{1.06}{0.06}(1 - 0.0969315) = 15.95421$, so the policy value is $0.0969315 - 0.004353264 \times 15.95421 = 0.02747861$. The surrender value of the policy is therefore $0.02747861 \times 0.8 = 0.02198289$. To find the term of the remaining policy, we need to solve

$$\begin{aligned} A_{48:\bar{t}|} &= 0.02198289 \\ A_{48} -_t {}_{t}p_{48} A_{48+t}(1.06)^{-t} &= 0.02198289 \\ {}_{t}p_{48} A_{48+t}(1.06)^{-t} &= 0.07494861 \\ A_{48+t} &= \frac{0.07494861(1.06)^{-t}}{{}_{t}p_{48}} \\ &= \frac{0.07494861(1.06)^{-t} \times 9907.10}{l_{48+t}} \end{aligned}$$

We evaluate the following values of $\frac{0.07494861(1.06)^{-t} \times 9907.10}{l_{48+t}}$:

t	$\frac{0.07494861(1.06)^{t} \times 9907.10}{l_{48+t}}$	t	$\frac{0.07494861(1.06)^{t} \times 9907.10}{l_{48+t}}$	t	$\frac{0.07494861(1.06)^{t} \times 9907.10}{l_{48+t}}$
		7	0.11365891	15	0.18462402
		8	0.12068350	16	0.19638280
1	0.07951905	9	0.12816250	17	0.20895895
2	0.08437510	10	0.13612831	18	0.22242071
3	0.08953585	11	0.14461668	19	0.23684293
4	0.09502181	12	0.15366607	20	0.25230953
5	0.10085479	13	0.16331890	21	0.26891372
6	0.10705864	14	0.17362132	22	0.28675959

We see that this first exceeds A_{48+t} when $t = 20$, so the new paid up term is 19 years.

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66
26	9997.00	9995.83	9994.40	9992.66
27	9995.14	9993.90	9992.38	9990.52
28	9993.16	9991.84	9990.22	9988.24
29	9991.05	9989.65	9987.92	9985.80
30	9988.81	9987.30	9985.46	9983.18
31	9986.40	9984.80	9982.82	9980.38
32	9983.83	9982.11	9979.99	9977.37
33	9981.07	9979.23	9976.95	9974.13
34	9978.11	9976.13	9973.68	9970.64
35	9974.93	9972.79	9970.16	9966.88
36	9971.50	9969.20	9966.36	9962.82
37	9967.80	9965.33	9962.25	9958.44
38	9963.81	9961.14	9957.82	9953.69
39	9959.50	9956.61	9953.02	9948.55
40	9954.84	9951.71	9947.82	9942.98
41	9949.79	9946.41	9942.19	9936.94
42	9944.32	9940.66	9936.08	9930.38
43	9938.39	9934.41	9929.45	9923.26
44	9931.96	9927.64	9922.25	9915.52
45	9924.97	9920.28	9914.42	9907.10
46	9917.37	9912.28	9905.91	9897.94
47	9909.11	9903.58	9896.65	9887.98
48	9900.13	9894.11	9886.57	9877.13
49	9890.36	9883.80	9875.59	9865.30
50	9879.71	9872.57	9863.63	9852.42
51	9868.12	9860.34	9850.59	9838.38
52	9855.48	9847.01	9836.39	9823.08
53	9841.72	9832.48	9820.90	9806.39
54	9826.71	9816.64	9804.02	9788.18
55	9810.34	9799.37	9785.60	9768.33
56	9792.49	9780.52	9765.51	9746.67
57	9773.03	9759.97	9743.60	9723.05
58	9751.79	9737.56	9719.69	9697.28
59	9728.63	9713.10	9693.62	9669.17
60	9703.36	9686.43	9665.17	9638.51
61	9675.80	9657.33	9634.15	9605.07
62	9645.73	9625.59	9600.31	9568.61
63	9612.94	9590.98	9563.42	9528.85
64	9577.18	9553.24	9523.19	9485.52
65	9538.19	9512.09	9479.35	9438.30
66	9495.69	9467.25	9431.58	9386.86
67	9449.37	9418.39	9379.54	9330.85
68	9398.90	9365.17	9322.87	9269.88
69	9343.95	9307.23	9261.20	9203.55
70	9284.12	9244.18	9194.11	9131.43
71	9219.03	9175.59	9121.17	9053.07
72	9148.24	9101.03	9041.91	8967.97
73	9071.30	9020.03	8955.85	8875.63
74	8987.73	8932.10	8862.49	8775.52
75	8897.04	8836.71	8761.27	8667.10
76	8798.69	8733.34	8651.66	8549.78
77	8692.13	8621.41	8533.09	8423.00
78	8576.81	8500.36	8404.95	8286.16
79	8452.13	8369.60	8266.68	8138.66
80	8317.52	8228.53	8117.67	7979.93
81	8172.36	8076.57	7957.35	7809.41
82	8016.08	7913.13	7785.15	7626.56
83	7848.11	7737.67	7600.54	7430.89
84	7667.89	7549.66	7403.05	7221.99
85	7474.92	7348.64	7192.27	6999.51
86	7268.77	7134.21	6967.86	6763.22
87	7049.07	6906.07	6729.62	6513.04
88	6815.55	6664.05	6477.46	6249.02
89	6568.09	6408.10	6211.48	5971.42
90	6306.70	6138.35	5931.96	5680.73
91	6031.59	5855.15	5639.41	5377.67
92	5743.19	5559.08	5334.61	5063.27
93	5442.15	5250.97	5018.61	4738.86
94	5129.44	4931.97	4692.79	4406.12
95	4806.33	4603.54	4358.89	4067.08
96	4474.39	4267.51	4018.96	3724.10
97	4135.60	3926.04	3675.44	3379.91
98	3792.25	3581.66	3331.11	3037.57
99	3447.02	3237.23	2989.05	2700.39
100	3102.90	2895.94	2652.63	2371.88
101	2763.19	2561.21	2325.37	2055.64
102	2431.39	2236.61	2010.90	1755.27
103	2111.15	1925.80	1712.81	1474.18
104	1806.12	1632.34	1434.48	1215.44
105	1519.82	1359.55	1178.94	981.65
106	1255.46	1110.36	948.70	774.71
107	1015.81	887.14	745.58	595.71
108	802.96	691.49	570.56	444.87
109	618.23	524.17	423.71	321.41
110	462.04	385.00	304.13	223.65
111	333.80	272.80	210.00	149.10
112	231.99	185.53	138.71	94.62
113	154.19	120.34	87.07	56.74
114	97.30	73.90	51.50	31.84
115	57.78	42.55	28.41	16.52
116	31.92	22.69	14.43	7.81
117	16.15	11.04	6.63	3.30
118	7.34	4.79	2.69	1.21
119	2.90	1.79	0.93	0.37
120	0.95	0.55	0.26	0.09
121	0.23	0.13	0.05	0.01
122	0.03	0.02	0.01	0.00