

ACSC/STAT 3720, Life Contingencies I

Winter 2018

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Homework Sheet 6

Due: Friday 23rd March: 12:30 PM

Basic Questions

1. An insurer issues 3,000 whole life insurance policies to standard lives aged 41. The appropriate interest rate is $i = 0.05$. The company calculates $A_{41} = 0.0989705$ and ${}^2A_{41} = 0.0190814$. If the death benefit is \$600,000, what annual premium should the company charge using the portfolio percentile method with a 90% probability of making a profit?
2. You are given the following values of A_x for various ages and interest rates:

x	i					
	0.03	0.0383523	0.04157806	0.042	0.042434	0.0453502
39	0.217774	0.148766	0.129115	0.126773	0.124417	0.109840
44	0.250364	0.177239	0.155883	0.153319	0.150733	0.134635
49	0.287221	0.210551	0.187599	0.184822	0.182017	0.164441
54	0.328587	0.249189	0.224839	0.221871	0.218869	0.199932
59	0.374567	0.293519	0.268073	0.264950	0.261784	0.241694

Using the lifetable in Table 1, and interest rate $i = 0.03$, calculate the net annual premium for a 10-year term insurance policy with benefit \$250,000, sold to a life aged 39, if:

- (a) The life works in a hazardous environment, and has mortality 0.012 higher than normal.
 - (b) The life is an impaired life, and is treated as a life five years older than its actual age.
3. An insurance company has a whole life insurance policy for an individual aged 47. The death benefit of this policy is \$350,000, and the interest rate is $i = 0.04$. Premiums are payable until age 80. The insurance company calculates $A_{47} = 0.184808$, and $A_{80} = 0.526062$. Therefore, the net annual premium for the policy is \$3,529.59. What is the policy value if the life survives to age 62? [Use the ultimate part of the lifetable in Table 1.]
 - (a) Using the same basis as the premium basis, which gives $A_{62} = 0.309353$.
 - (b) Using the reserve basis $i = 0.03$, which gives $A_{80} = 0.609835$, $A_{62} = 0.404336$, $A_{47} = 0.271948$, $\ddot{a}_{80} = 13.39566$, $\ddot{a}_{62} = 20.45113$ and $\ddot{a}_{47} = 24.99645$.
 4. A life aged 47 takes out a whole life insurance with benefit \$400,000. The initial cost of this insurance is \$3,000 plus 30% of the first annual premium. The renewal cost is 3% of each subsequent premium. The interest rate is $i = 0.06$. Using the ultimate part of the lifetable in Table 1, we can calculate $A_{47} = 0.0921683$.
 - (a) Calculate the gross premium for this policy.
 - (b) Calculate the gross policy value after 2 years.

Standard Questions

5. A life insurance company sells 10-year term insurance policies to lives aged 53 for whom the Ultimate part of the lifetable in Table 1 is appropriate. The death benefit is \$400,000.

The interest rate is $i = 0.06$. This gives $A_{53} = 0.124241$ and $A_{63} = 0.199371$, and also ${}^2A_{53} = 0.0311226$ and ${}^2A_{63} = 0.0655426$. Using the portfolio premium principle with a 95% probability of profit, they want to ensure the annual premium is under \$2000. How many policies do they need to include in the portfolio to ensure this?

6. A life insurance company sells whole life insurance policies to lives aged 43. The interest rate is $i = 0.05$. Under the equivalence principle, the annual premium for a policy with benefit \$100,000 would be \$367.42. Using the portfolio premium percentile principle on a portfolio of 2,000 policies with benefit \$100,000, and a 95% probability of profit, they calculate a premium of \$394.03. What premium would they calculate if they used a portfolio of 1,600 policies with benefit \$100,000 and 200 policies with benefit \$200,000 (assume that premium is proportional to benefit, so these policies would pay double to premium of the others), and a 90% probability of profit.

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$	x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66	74	8987.73	8932.10	8862.49	8775.52
26	9997.00	9995.83	9994.40	9992.66	75	8897.04	8836.71	8761.27	8667.10
27	9995.14	9993.90	9992.38	9990.52	76	8798.69	8733.34	8651.66	8549.78
28	9993.16	9991.84	9990.22	9988.24	77	8692.13	8621.41	8533.09	8423.00
29	9991.05	9989.65	9987.92	9985.80	78	8576.81	8500.36	8404.95	8286.16
30	9988.81	9987.30	9985.46	9983.18	79	8452.13	8369.60	8266.68	8138.66
31	9986.40	9984.80	9982.82	9980.38	80	8317.52	8228.53	8117.67	7979.93
32	9983.83	9982.11	9979.99	9977.37	81	8172.36	8076.57	7957.35	7809.41
33	9981.07	9979.23	9976.95	9974.13	82	8016.08	7913.13	7785.15	7626.56
34	9978.11	9976.13	9973.68	9970.64	83	7848.11	7737.67	7600.54	7430.89
35	9974.93	9972.79	9970.16	9966.88	84	7667.89	7549.66	7403.05	7221.99
36	9971.50	9969.20	9966.36	9962.82	85	7474.92	7348.64	7192.27	6999.51
37	9967.80	9965.33	9962.25	9958.44	86	7268.77	7134.21	6967.86	6763.22
38	9963.81	9961.14	9957.82	9953.69	87	7049.07	6906.07	6729.62	6513.04
39	9959.50	9956.61	9953.02	9948.55	88	6815.55	6664.05	6477.46	6249.02
40	9954.84	9951.71	9947.82	9942.98	89	6568.09	6408.10	6211.48	5971.42
41	9949.79	9946.41	9942.19	9936.94	90	6306.70	6138.35	5931.96	5680.73
42	9944.32	9940.66	9936.08	9930.38	91	6031.59	5855.15	5639.41	5377.67
43	9938.39	9934.41	9929.45	9923.26	92	5743.19	5559.08	5334.61	5063.27
44	9931.96	9927.64	9922.25	9915.52	93	5442.15	5250.97	5018.61	4738.86
45	9924.97	9920.28	9914.42	9907.10	94	5129.44	4931.97	4692.79	4406.12
46	9917.37	9912.28	9905.91	9897.94	95	4806.33	4603.54	4358.89	4067.08
47	9909.11	9903.58	9896.65	9887.98	96	4474.39	4267.51	4018.96	3724.10
48	9900.13	9894.11	9886.57	9877.13	97	4135.60	3926.04	3675.44	3379.91
49	9890.36	9883.80	9875.59	9865.30	98	3792.25	3581.66	3331.11	3037.57
50	9879.71	9872.57	9863.63	9852.42	99	3447.02	3237.23	2989.05	2700.39
51	9868.12	9860.34	9850.59	9838.38	100	3102.90	2895.94	2652.63	2371.88
52	9855.48	9847.01	9836.39	9823.08	101	2763.19	2561.21	2325.37	2055.64
53	9841.72	9832.48	9820.90	9806.39	102	2431.39	2236.61	2010.90	1755.27
54	9826.71	9816.64	9804.02	9788.18	103	2111.15	1925.80	1712.81	1474.18
55	9810.34	9799.37	9785.60	9768.33	104	1806.12	1632.34	1434.48	1215.44
56	9792.49	9780.52	9765.51	9746.67	105	1519.82	1359.55	1178.94	981.65
57	9773.03	9759.97	9743.60	9723.05	106	1255.46	1110.36	948.70	774.71
58	9751.79	9737.56	9719.69	9697.28	107	1015.81	887.14	745.58	595.71
59	9728.63	9713.10	9693.62	9669.17	108	802.96	691.49	570.56	444.87
60	9703.36	9686.43	9665.17	9638.51	109	618.23	524.17	423.71	321.41
61	9675.80	9657.33	9634.15	9605.07	110	462.04	385.00	304.13	223.65
62	9645.73	9625.59	9600.31	9568.61	111	333.80	272.80	210.00	149.10
63	9612.94	9590.98	9563.42	9528.85	112	231.99	185.53	138.71	94.62
64	9577.18	9553.24	9523.19	9485.52	113	154.19	120.34	87.07	56.74
65	9538.19	9512.09	9479.35	9438.30	114	97.30	73.90	51.50	31.84
66	9495.69	9467.25	9431.58	9386.86	115	57.78	42.55	28.41	16.52
67	9449.37	9418.39	9379.54	9330.85	116	31.92	22.69	14.43	7.81
68	9398.90	9365.17	9322.87	9269.88	117	16.15	11.04	6.63	3.30
69	9343.95	9307.23	9261.20	9203.55	118	7.34	4.79	2.69	1.21
70	9284.12	9244.18	9194.11	9131.43	119	2.90	1.79	0.93	0.37
71	9219.03	9175.59	9121.17	9053.07	120	0.95	0.55	0.26	0.09
72	9148.24	9101.03	9041.91	8967.97	121	0.23	0.13	0.05	0.01
73	9071.30	9020.03	8955.85	8875.63	122	0.03	0.02	0.01	0.00