ACSC/STAT 4703, Actuarial Models II
Fall 2015
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Homework Sheet 4
Due: Wednesday 28th October: 10:30 PM

Basic Questions

1. An insurance company models number of claims an individual makes in a year as following a negative binomial distribution with $\beta = 1.4$, and R an unknown parameter with prior distribution a gamma distribution with $\alpha = 3$ and $\theta = 0.04$.

   (a) What is the probability that a random individual makes exactly 3 claims?

   (b) The company now observes the following claim frequencies:

Number of claims	Frequency
0	584
1	90
2	36
3	12
4	3
5	3
6	1

   What is the probability that $R > 0.4$? [You may use numerical integration to calculate this.]

   (c) Calculate the predictive probability that an individual makes 5 claims next year. [You may use numerical integration to calculate this.]

2. An insurance company models loss sizes as following a Pareto distribution with $\alpha = 3$, and finds that the posterior distribution for Θ is a Gamma distribution with $\alpha = 4$ and $\theta = 1400$. Calculate the Bayes estimate for Θ based on a loss function:

   (a) $l(\hat{\theta}, \theta) = (\hat{\theta} - \theta)^2$

   (b) $l(\hat{\theta}, \theta) = (\hat{\theta} - \theta)^4$

3. An insurance company models claim amounts as following an exponential distribution with mean Θ, where the prior distribution for Θ is a Gamma distribution with $\alpha = 701$ and $\theta = 600$. They observe 700 claims, with mean claim amount \$3,742. Calculate a 95% credibility interval for Θ.

   (a) Using an HPD interval. [For performing the integral, make the substitution $z = \frac{\theta}{600} + \frac{2619400}{\theta}$. There are two values of θ for any particular value of z, and if these values are $\theta_1 < \theta_2$, then we have that

$$\left.\frac{dz}{d\theta}\right|_{\theta_2} - \left.\frac{dz}{d\theta}\right|_{\theta_1} = \frac{2619400}{180000z^2-1571640000-600z\sqrt{90000z^2-1571640000}} - \frac{2619400}{180000z^2-1571640000+600z\sqrt{90000z^2-1571640000}}$$

   You can solve the required integral numerically.]

(b) With equal probability above and below the interval. [You may use numerical integration to calculate this.]

4. Calculate a conjugate prior distribution for the variance of a normal distribution with mean 0.

Standard Questions

5. An insurance company models number of claims made by an individual in a year as following a Poisson distribution and finds that the posterior distribution for Λ is a Gamma distribution with $\alpha = 4$ and $\theta = 0.02$. The company decides to use an estimate $\hat{\lambda}$ such that the probability of 3 or more claims using $\hat{\lambda}$ is the same as the probability of 3 or more claims under the predictive distribution.

Bonus Question

6. An insurance company models loss amounts as following a Weibull distribution with $\tau = 3$. It uses the inverse gamma prior for the unknown parameter Θ, with parameters $\alpha = 3$ and $\theta = 6000$. This is a conjugate prior, and the posterior distribution after observing N observations $X_1, \ldots, X_n$ with $\sum_{i=1}^{n} X_i{}^{\tau} = t$ is inverse gamma with $\alpha = 3 + N$, and $\theta = \frac{1}{\frac{1}{6000}+t}$. Calculate the probability that the posterior probability of $\Theta > 3000$ is more than 0.05, after a sample of 10 observations.