ACSC/STAT 4703, Actuarial Models II
Fall 2016
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Homework Sheet 1
Due: Friday 30th September: 10:30 PM

Basic Questions

1. Loss amounts follow a gamma distribution with $\alpha = 3$ and $\theta = 8,000$. The distribution of the number of losses is given in the following table:

Number of Losses	Probability
0	0.04
1	0.43
2	0.34
3	0.19

Assume all losses are independent and independent of the number of losses. The insurance company buys excess-of-loss reinsurance on the part of the loss above $100,000. Calculate the expected payment for this excess-of-loss reinsurance.

2. An insurance company models loss frequency as negative binomial with $r = 2$, $\beta = 6$, and loss severity as pareto with $\alpha = 6$ and $\theta = \$30,000$. Calculate the expected aggregate payments if there is a policy limit of $100,000 and a deductible of $5,000 applied to each claim.

3. Aggregate payments have a compund distribution. The frequency distribution is negative binomial with $r = 5$ and $\beta = 5$. The severity distribution is a Pareto distribution with $\alpha = 5$ and $\theta = 16000$. Use a Gamma approximation to aggregate payments to estimate the probability that aggregate payments are more than $150,000.

4. Claim frequency follows a negative binomial distribution with $r = 4$ and $\beta = 4.4$. Claim severity (in thousands) has the following distribution:

Severity	Probability
1	0.6
2	0.3
3	0.06
4	0.03
5	0.006
6 or more	0.003

Use the recursive method to calculate the exact probability that aggregate claims are at least 6.

5. Use an arithmetic distribution ($h = 1$) to approximate a Pareto distribution with $\alpha = 6$ and $\theta = 30$.

   (a) Using the method of rounding, calculate the mean of the the arithmetic approximation, conditional on lying in the interval 4.5 and 7.5. (That is, calculate $\mathbb{E}(X|4.5 < X < 5.5)$, where X follows the arithmetic distribution used to approximate.)

   (b) Using the method of local moment matching, matching 1 moment on each interval, estimate the probability that the value lies between 4.5 and 7.5.

Standard Questions

6. The number of claims an insurance company receives follows a Poisson distribution with $\lambda = 84$. Claim severity follows a negative binomial distribution with $r = 21$ and $\beta = 1.8$. Calculate the probability that aggregate losses exceed \$4000.

   (a) Starting the recurrence 6 standard deviations below the mean.

   (b) Using a suitable convolution.