ACSC/STAT 4703, Actuarial Models II

Fall 2016

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Homework Sheet 8

Due: Friday 2nd December: 10:30 PM

For each question that asks you to simulate a small number of samples from a distribution, use the following simulated uniform values, starting from the first, and using as many numbers as needed for the question. Go back to the first value at the start of each part question.

```
0.84099888 0.70862315 0.77035929 0.10697185 0.50703901 0.33571753
0.97884700 0.82590460 0.32134825 0.17141305 0.06130620 0.64046291
0.20002273 0.86746587 0.46627231 0.53668803 0.41593316 0.86867032
0.81685865 0.88962299 0.90098504 0.01358413 0.80805736 0.17980851
0.29944468 0.35325712 0.57054569 0.22218065 0.72032510 0.17402895
0.08618214 0.86043490 0.83286257 0.75478531 0.60834949 0.08403240
0.81798767 0.19805759 0.34754212 0.36734934
```

Basic Questions

1. Use the method of inversion to simulate two random samples from

   (a) a Weibull distribution with $\tau = 3$, $\theta = 70$.

   (b) a Pareto distribution with $\alpha = 4$, $\theta = 1200$.

2. An insurance company classifies individuals into three classes, each with a different claim frequency distribution, as shown in the following table:

Class	Probability	Frequency Distribution	Parameters
1	0.60	Binomial	$n = 29, p = 0.046$
2	0.25	Poisson	$\lambda = 0.08$
3	0.15	Poisson	$\lambda = 0.18$

   (a) Simulate 3 claim frequencies from 3 random individuals.

   (b) Simulate 3 claim frequencies from a single individual.

3. A home insurance policy has three types of claim with probabilities in the table below:

Claim Type	Probability
Fire	0.07
Theft	0.55
Other	0.38

   Simulate the number of each type from a sample of 744 claims.

4. Use a stochastic process method to simulate 3 samples from each of the following distributions:

   (a) A binomial distribution with $n = 8$ and $p = 0.08$.

   (b) A negative binomial distribution with $r = 3$ and $\beta = 1.1$.

5. Simulate 4 samples from a normal distribution with $\mu = -3$ and $\sigma = 2$ using

   (a) A Box-Muller transformation.

   (b) The polar method.

Standard Questions

6. An insurance company models total claim frequency as following a Poisson distribution with $\lambda = 4.2$. Claim severity is independent of frequency and follows a Weibull distribution with $\tau = 0.6$ and $\theta = \$2,100$. They calculate the mean of the aggregate claim distribution as $13,270.36 and the variance as 171523504.582. They are interested in how much would be saved by introducing a deductible of $2000 per claim and a policy limit of $10,000 per claim, with a maximum out of pocket cost of $5000. (This means that once the policyholder has paid a total of $5,000, either from deductibles or from losses exceeding the policy limit, all future deductibles and policy limits are waived). They will use a simulation to calculate the new mean aggregate claims.

   (a) What sample size should they use so that the relative error in the estimated mean is less than 1% with probability 0.95 [You may assume that the new mean aggregate claims are close to the current mean of $13,270.36, for the purposes of determining the size of the 1% relative error.]

   (b) Use the random numbers at the top of this sheet to simulate the first 5 aggregate losses with the new modifications.

7. An insurance company is estimating the VaR of a new policy. They simulate 100 aggregate losses, and estimate the following VaR at various levels:

level	VaR
92%	$71,300
93%	$71,875
94%	$72,125
95%	$72,500
96%	$72,975
97%	$73,225
98%	$73,750
99%	$74,750

Based on these estimates, how many aggregate losses do the need to simulate, so that the probability of an error greater than 1% in their VaR estimate at the 95% level is at most 0.01? [Hint: to obtain a confidence interval for the VaR, consider instead estimating the percentile of a given point — what is the distribution of the sample percentile of the true VaR?]