ACSC/STAT 4703, Actuarial Models II
Fall 2017
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Homework Sheet 3
Model Solutions

Basic Questions

1. *A homeowner's house is valued at \$420,000, but is insured at \$270,000. The insurer requires 75% coverage for full insurance. The home sustains \$3,100 from a fire. The policy has a deductible of \$2,000, which decreases linearly to zero when the total cost of the loss is \$6,000. How much does the insurance company reimburse?*

   The proportion of coinsurance is $\frac{270000}{420000\times 0.75} = \frac{6}{7}$The deductible is $2000\frac{6000-3100}{6000-2000} = \$1,450$. The insurance therefore pays $(3100 - 1450) \times \frac{6}{7} = \frac{990}{7} = \$141.43$.

2. *A homeowners insurance company has three types of coverages with different expected loss ratios, has the following data on recent claims:*

Policy Type	*Policy Year*	*Earned Premiums*	*Expected Loss Ratio*	*Losses paid to date*
Homeowner's insurance	*2014*	*\$400,000*	*0.72*	*\$270,000*
	2015	*\$480,000*	*0.72*	*\$130,000*
	2016	*\$590,000*	*0.74*	*\$70,000*
Tennant's insurance	*2014*	*\$70,000*	*0.83*	*\$58,600*
	2015	*\$72,000*	*0.83*	*\$44,300*
	2016	*\$75,000*	*0.83*	*\$29,400*
Fire insurance	*2014*	*\$300,000*	*0.65*	*\$126,000*
	2015	*\$350,000*	*0.65*	*\$85,000*
	2016	*\$380,000*	*0.67*	*\$17,000*

   Calculate the loss reserves at the end of 2016.

   We use the expected loss ratios to calculate the expected total payments for each policy year:

Policy Type	Policy Year	Expected Payments	Losses paid to date	Reserves needed
Homeowner's insurance	2014	\$288,000	\$270,000	\$18,000
	2015	\$345,600	\$130,000	\$215,600
	2016	\$436,600	\$70,000	\$366,600
Tennant's insurance	2014	\$58,100	\$58,600	\$0
	2015	\$59,760	\$44,300	\$15,460
	2016	\$62,250	\$29,400	\$32,850
Fire insurance	2014	\$195,000	\$126,000	\$69,000
	2015	\$227,500	\$85,000	\$142,500
	2016	\$254,600	\$17,000	\$237,600

The total reserves are therefore \$1,097,610.

3. *The following table shows the paid losses on claims from one line of business of an insurance company over the past 6 years.*

		Development year					
Accident year	*Earned premiums*	*0*	*1*	*2*	*3*	*4*	*5*
2011	*3,156*	*870*	*95*	*253*	*727*	−425	*851*
2012	*3,930*	*844*	*184*	*709*	*409*	*300*	
2013	*3,248*	*1,394*	*258*	*184*	−3		
2014	*4,955*	*1,291*	*54*	*856*			
2015	*4,142*	*1,422*	*579*				
2016	*4,806*	*1,754*					

Assume that all payments on claims arising from accidents in 2011 have now been settled. Estimate the future payments arising each year from open claims arising from accidents in each calendar year using

(a) The loss development triangle method

We first calculate the cumulative loss development:

	Development year					
Accident year	0	1	2	3	4	5
2011	870	965	1,218	1,945	1,520	2,371
2012	844	1,028	1,737	2,146	2,446	
2013	1,394	1,652	1,836	1,833		
2014	1,291	1,345	2,201			
2015	1,422	2,001				
2016	1,754					

The loss development factors are therefore given by

$\frac{6991}{5821} = 1.20099639237$

$\frac{6992}{4990} = 1.40120240481$

$\frac{5924}{4791} = 1.23648507618$

$\frac{3966}{4091} = 0.969445123442$

$\frac{2371}{1520} = 1.55986842105$

We use these factors to estimate the following developed losses

	Development year					
Accident year	0	1	2	3	4	5
2011						2,371
2012					2,446	3,815
2013				1,833	1,777	2,772
2014			2,201	2,722	2,638	4,115
2015		2,001	2,804	3,467	3,361	5,243
2016	1,754	2,107	2,952	3,650	3,538	5,519

If we use the average, the loss development factors are:

$\frac{1}{5}\left(\frac{965}{870}+\frac{1028}{844}+\frac{1652}{1394}+\frac{1345}{1291}+\frac{2001}{1422}\right)=1.19225696533$

$\frac{1}{4}\left(\frac{1218}{965}+\frac{1737}{1028}+\frac{1836}{1652}+\frac{2201}{1345}\right)=1.42491906345$

$\frac{1}{3}\left(\frac{1945}{1218}+\frac{2146}{1737}+\frac{1833}{1836}\right)=1.27690319572$

$\frac{1}{2}\left(\frac{1520}{1945}+\frac{2446}{2146}\right)=0.96064298498$

$\frac{2371}{1520}=1.55986842105$

(b) The Bornhuetter-Ferguson method with expected loss ratio 0.76.

Using an expected loss ratio of 0.76, the expected ultimate losses are:

Accident year	Expected ultimate losses
2011	2398.56
2012	2986.80
2013	2468.48
2014	3765.80
2015	3147.92
2016	3652.56

Using these and the loss development factors calculated above, the expected payments are

	Development year				
Accident year	1	2	3	4	5
2011	153.21	367.29	303.36	−48.46	860.89
2012	190.79	457.37	377.75	−60.35	1072.02
2013	157.68	378.00	312.20	−49.88	885.99
2014	240.55	576.66	476.28	−76.09	1351.62
2015	201.08	482.04	398.13	−63.61	1129.85
2016	233.32	559.32	461.96	−73.80	1310.98

Using the average to calculate loss development factors, the proportion of total losses paid in each year is

Development year	Proportion of losses
0	0.307632333161
1	0.059144458813
2	0.155850450938
3	0.144717153734
4	−0.026264683444
5	0.358920286798

	Development year				
Accident year	1	2	3	4	5
2011					
2012					1072.02
2013				−64.83	885.99
2014			544.98	−98.91	1351.62
2015		490.60	455.56	−82.68	1129.85
2016	216.03	569.25	528.59	−95.93	1310.98

4. An actuary is reviewing the following claims data:

No. of closed claims

Acc. Year	*0*	*1*	*2*	*3*	*4*	*Ult.*
2012	*250*	*335*	*370*	*395*	*400*	*400*
2013	*280*	*385*	*400*	*450*		*460*
2014	*330*	*395*	*470*			*500*
2015	*320*	*460*				*540*
2016	*360*					*580*

Total paid losses on closed claims (000's)

Acc. Year	*0*	*1*	*2*	*3*	*4*
2012	*723*	*2,087*	*2,263*	*2,822*	*4,783*
2013	*1,509*	*2,641*	*2,948*	*5,256*	
2014	*1,745*	*3,214*	*3,754*		
2015	*3,094*	*3,244*			
2016	*2,824*				

(a) Calculate tables of percentage of claims closed and cumulative average losses.

Percentage of claims closed

Acc. Year	0	1	2	3	4
2012	62.5	83.75	92.5	98.75	100
2013	60.87	83.70	86.96	97.83	
2014	66	79	94		
2015	59.26	85.19			
2016	62.07				

Average paid losses per claim on closed claims (000's)

Acc. Year	0	1	2	3	4
2012	2,892	6,230	6,116	7,144	11,958
2013	5,389	6,860	7,370	11,680	
2014	5,288	8,137	7,987		
2015	9,669	7,052			
2016	7,844				

(b) Adjust the total loss table to use the current disposal rate.

The current disposal rate is given by the last number in each row in the percentage of claims closed table — that is

Acc. Year	0	1	2	3	4
Disposal rate	62.07	85.19	94	97.83	100

We therefore adjust each entry in the total claims table by multiplying by the following factors:

Acc. Year	0	1	2	3	4
2012	$\frac{62.07}{62.5}$	$\frac{85.19}{83.75}$	$\frac{94}{92.5}$	$\frac{97.83}{98.75}$	$\frac{100}{100}$
2013	$\frac{62.07}{60.87}$	$\frac{85.19}{83.70}$	$\frac{94}{86.96}$	$\frac{97.83}{97.83}$	
2014	$\frac{62.07}{66}$	$\frac{85.19}{79}$	$\frac{94}{94}$		
2015	$\frac{62.07}{59.26}$	$\frac{85.19}{85.19}$			
2016	$\frac{62.07}{62.07}$				

The adjusted payments are:

Acc.	Development Year				
Year	0	1	2	3	4
2012	718	2,123	2,300	2,796	4,783
2013	1,539	2,688	3,187	5,256	
2014	1,641	3,466	3,754		
2015	3,241	3,244			
2016	2,824				

(c) Use the chain ladder method to estimate claim development based on the adjusted numbers. Compare this to the chain ladder method on aggregate payments on closed claims.

For the adjusted payments, the loss-development factors are

$\frac{11521}{7139} = 1.61381145819$

$\frac{9241}{8277} = 1.11646731908$

$\frac{8052}{5487} = 1.46746856206$

$\frac{4783}{2796} = 1.71065808298$

These result in the following total cumulative loss payments:

Acc.	Development Year				
Year	0	1	2	3	4
2013					8991.22
2014				5508.88	9423.80
2015			3621.82	5314.91	9091.99
2016		4557.40	5088.19	7466.76	12773.08

For un-adjusted payments on closed claims, the development factors are

$\frac{11186}{7071} = 1.58195446189$

$\frac{8965}{7942} = 1.12880886427$

$\frac{8078}{5211} = 1.55018230666$

$\frac{4783}{2822} = 1.694897236$

These result in the following total cumulative loss payments

Acc.	Development Year				
Year	0	1	2	3	4
2013					8908.38
2014				5819.38	9863.26
2015			3661.86	5676.54	9621.16
2016		4467.44	5042.89	7817.39	13249.68

Using the average, the loss development factors on adjusted data are:

1.95411639968, 1.11736845646, 1.4324260242, 1.71065808298

So the esimated claim development is

Acc.	Development Year				
Year	0	1	2	3	4
2013					8991
2014				5377	9199
2015			3625	5192	8882
2016		5518	6166	8833	15109

Using the unadjusted data, the loss development factors are:

1.88176602337, 1.12286345271, 1.51496044863, 1.694897236 So the esimated claim development is

Acc.	Development Year				
Year	0	1	2	3	4
2013					8908
2014				5687	9639
2015			3643	5518	9353
2016		5314	5967	9040	15322

Standard Questions

5. *An insurance company insures 10,000 homes. Each home makes a claim with probability 0.02. If a home makes a claim, the loss distribution of the claim is a mixture distribution: with probability 0.95, the loss amount follows an exponential distribution with mean $5,000. With probability 0.05, the loss amount follows an exponential distribution with mean $300,000. The insurance company sets its premium at 110% of expected claims. What policy limit should it set to ensure that the probability that aggregate claims exceed aggregate premiums is less than 0.001? [Note that changes to the policy limit will change the premium.]*

   For an exponential random variable X with mean θ, the expectation of the limited loss random variable $X \wedge u$ is given by $\mathbb{E}(X \wedge u) = \int_0^u e^{-\frac{x}{\theta}}\,dx = \theta(1-e^{-\frac{u}{\theta}})$. If the policy limit is u, then the expected claim per policy is

$$0.02\left(0.95 \times 5000(1-e^{-\frac{u}{5000}}) + 0.05 \times 30000(1-e^{-\frac{u}{30000}}\right) = 125-95e^{-\frac{u}{5000}}-30e^{-\frac{u}{30000}}$$

   The premium is therefore 110% of this, which is $137.5 - 104.5e^{-\frac{u}{5000}} - 33e^{-\frac{u}{30000}}$. The variance of the limited loss random variable (per loss) is given by

$$
\begin{aligned}
\mathbb{E}\left((X\wedge u)^2\right) &= \int_0^u x^2\left(\frac{0.95}{5000}e^{-\frac{x}{5000}}+\frac{0.05}{30000}e^{-\frac{x}{30000}}\right)\,dx+\left(0.95e^{-\frac{u}{5000}}+0.05e^{-\frac{u}{30000}}\right)u^2\\
&= \left[-x^2\left(0.95e^{-\frac{x}{5000}}+0.05e^{-\frac{x}{30000}}\right)\,dx\right]_0^u+2\int_0^u x\left(0.95e^{-\frac{x}{5000}}+0.05e^{-\frac{x}{30000}}\right)\,dx\\
&\quad +\left(0.95e^{-\frac{u}{5000}}+0.05e^{-\frac{u}{30000}}\right)u^2\\
&= 2\left(\left[-x\left(4750e^{-\frac{x}{5000}}+1500e^{-\frac{x}{30000}}\right)\right]_0^u+\int_0^u\left(4750e^{-\frac{x}{5000}}+1500e^{-\frac{x}{30000}}\right)\,dx\right)\\
&= -2u\left(4750e^{-\frac{u}{5000}}+1500e^{-\frac{u}{30000}}\right)+137500000-47500000e^{-\frac{x}{5000}}-90000000e^{-\frac{x}{30000}}\\
\mathrm{Var}(X\wedge u) &= 137500000-2u\left(4750e^{-\frac{u}{5000}}+1500e^{-\frac{u}{30000}}\right)-47500000e^{-\frac{x}{5000}}-90000000e^{-\frac{x}{30000}}\\
&\quad -\left(6250-4750e^{-\frac{u}{5000}}-1500e^{-\frac{u}{30000}}\right)^2\\
&= 98437500-9500ue^{-\frac{u}{5000}}-3000ue^{-\frac{u}{30000}}+11875000e^{-\frac{u}{5000}}-71250000e^{-\frac{u}{30000}}\\
&\quad -14250000e^{-u\left(\frac{1}{5000}+\frac{1}{30000}\right)}-22562500e^{-\frac{2u}{5000}}-2250000e^{-\frac{2u}{30000}}
\end{aligned}
$$

Now the variance per policy is

$$
\begin{aligned}
\mathrm{Var}(P) &= 0.02\,\mathrm{Var}(X\wedge u)+0.02\times 0.98\left(\mathbb{E}(X\wedge u)\right)^2\\
&= 0.02\mathbb{E}((X\wedge u)^2)-0.02^2\left(\mathbb{E}(X\wedge u)\right)^2\\
&= 2734375-190ue^{-\frac{u}{5000}}-60ue^{-\frac{u}{30000}}-926250e^{-\frac{u}{5000}}-1792500e^{-\frac{u}{30000}}\\
&\quad -5700e^{-u\left(\frac{1}{5000}+\frac{1}{30000}\right)}-9025e^{-\frac{2u}{5000}}-900e^{-\frac{2u}{30000}}
\end{aligned}
$$

The variance of the average loss per policy is obtained by dividing this by 10000. Using a normal approximation, the probability that aggregate losses exceed premiums is

$$
1-\Phi\left(\sqrt{10000}\frac{12.5-9.5e^{-\frac{u}{5000}}-3e^{-\frac{u}{30000}}}{\sqrt{\mathrm{Var}(P)}}\right)
$$

We want to solve

$$
\begin{aligned}
\Phi\left(\sqrt{10000}\frac{12.5-9.5e^{-\frac{u}{5000}}-3e^{-\frac{u}{30000}}}{\sqrt{\mathrm{Var}(P)}}\right) &= 0.999\\
\sqrt{10000}\frac{12.5-9.5e^{-\frac{u}{5000}}-3e^{-\frac{u}{30000}}}{\sqrt{\mathrm{Var}(P)}} &= 3.090232\\
(12.5-9.5e^{-\frac{u}{5000}}-3e^{-\frac{u}{30000}})^2 &= 0.00009549536\,\mathrm{Var}(P)\\
0.01009549536(125-95e^{-\frac{u}{5000}}-30e^{-\frac{u}{30000}})^2 &= 0.00009549536\times 0.02\mathbb{E}((X\wedge u)^2)
\end{aligned}
$$

$$105.7171(125 - 95e^{-\frac{u}{5000}} - 30e^{-\frac{u}{30000}})^2 = -2u\left(95e^{-\frac{u}{5000}} + 30e^{-\frac{u}{30000}}\right) + 2750000 - 950000e^{-\frac{x}{5000}} - 18000$$

Solving this numerically gives $u = \$27,195$.