ACSC/STAT 4703, Actuarial Models II
Fall 2017
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Homework Sheet 6
Model Solutions

Basic Questions

1. *An insurance company starts a new line of insurance in 2016, and collects a total of \$1,600,000 in premiums that year, and the estimated incurred losses for accident year 2016 are \$684,000. The premium payments are uniformly distributed over the year. An actuary is using this data to estimate rates for premium year 2018. Claims are subject to 4% inflation per year. By what percentage should premiums increase from 2016 in order to achieve a loss ratio of 0.75.*

 Assuming the 2016 premiums are uniformly distributed over the year, the earned premiums from 2016 are \$800,000, so the loss ratio is $\frac{684000}{800000} = 0.855$. To achieve a loss ratio of 0.75 in 2016, the premium would need to be increased by a factor of $\frac{0.855}{0.75} = 1.14$. This is for losses uniformly distributed over 2016. The premium for policy year 2018 applies to 1-year policies with starting date uniformly distributed over 2018. The inflation from the start of 2016 to a date uniformly distributed over 2016 is $\int_0^1 (1.04)^t\,dt = \left[\frac{1.04^t}{\log(1.04)}\right]_0^1 = \frac{0.04}{\log(1.04)} = 1.01986926764$. The inflation from the start of 2018 to a date distributed between 2018 and 2019 with density function $\begin{cases} t & \text{if } 0 < t < 1 \\ 2-t & \text{if } 1 \leqslant t < 2 \end{cases}$ is

$$\begin{aligned}
\int_0^1 t(1.04)^t\,dt + 1.04\int_0^1 (1-t)(1.04)^t\,dt &= 1.04\int_0^1 (1.04)^t\,dt - 0.04\int_0^1 t(1.04)^t \\
&= \frac{1.04\times 0.04}{\log(1.04)} - 0.04\left(\left[t\frac{1.04^t}{\log(1.04)}\right]_0^1 - \int_0^1 \frac{1.04^t}{\log(1.04)}\,dt\right) \\
&= \frac{1.04\times 0.04}{\log(1.04)} - 0.04\left(\frac{1.04}{\log(1.04)} - \frac{0.04}{\log(1.04)^2}\,dt\right) \\
&= \frac{0.04^2}{\log(1.04)^2} \\
&= 1.04013332308
\end{aligned}$$

 The inflation from a time uniformly distributed over accident year 2016 to a time uniformly distributed over policy year 2018 is therefore $\frac{1.04^2\times 1.04013332308}{1.01986926764} = 1.10309059988$. The increase in premium needed is therefore a factor of $1.10309059988 \times 1.14 = 1.25752328386$, or an increase of 25.75%.

2. *An insurer collects \$340,000 in earned premiums for accident year 2016. The total loss payments are \$284,000. Payments are subject to inflation of 3%, and policies are sold uniformly throughout the year. If the insurer's permissible loss ratio is 75%, by how much should the premium be changed for policy year 2018?*

   The loss ratio for 2016 was $\frac{284000}{340000} = 0.835294117647$. With inflation of 3%, the inflation from the start of 2016 to a uniformly distributed time during the year is $\int_0^1 1.03^t\,dt = \frac{0.03}{\log(1.03)} = 1.01492610407$. As in Question 1, the inflation from the start of 2016 to a random loss associated with policy year 2018 is $(1.03)^2\left(\frac{0.03}{\log(1.03)}\right)^2 = 1.09280656402$. The percentage change in premium for policy year 2018 is therefore $\frac{0.835294117647}{0.75} \times \frac{1.09280656402}{1.01492610407} - 1 =$ 19.92% increase.

3. *An auto insurer classifies policies into three age groups — young, medium and old. The experience from policy year 2016 is:*

Age Class	*Current differential*	*Earned premiums*	*Loss payments*
Young	*1.54*	*3,300*	*1,100*
Medium	*1*	*4,600*	*3,900*
Old	*0.89*	*2,700*	*1,400*

   The base premium was \$580. Claim amounts are subject to 4% annual inflation. If the expense ratio is 20%, calculate the new premiums for each age class for policy year 2018.

   The overall loss ratio was $\frac{6400}{10600} = 0.603773584906$. Using the loss ratio method, the new differentials are

Age Class	Current differential	Loss ratio	New differential
Young	1.54	$\frac{1100}{3300} = 0.333333333333$	$1.54 \times \frac{0.333333333333}{0.847826086957} = 0.605470085469$
Medium	1	$\frac{3900}{4600} = 0.847826086957$	1
Old	0.89	$\frac{1400}{2700} = 0.518518518519$	$0.89 \times \frac{0.518518518519}{0.847826086957} = 0.544311490979$

   If we adjust the earned premiums to use these differentials, the earned premiums in 2016 for young drivers would be $1100 \times 0.847826086957 = 932.608695653$ and for old drivers would be $1400 \times 0.847826086957 = 1186.95652174$. The overall loss ratio would then be 0.847826086957. To get a loss ratio of 0.8 we would have to multiply the base premium by $\frac{0.847826086957}{0.8} = 1.0597826087$. In addition, we need to adjust for inflation by multiplying by 1.04^2. This means that the base premium should be $580 \times 1.04^2 \times 1.0597826087 = \$664.83$. The differentials are 0.605470085469 and 0.544311490979 so the premium for young drivers would be $0.605470085469 \times 664.831304351 = \$402.54$ and the premium for old drivers should be $0.544311490979 \times 664.831304351 = \$361.88$.

Standard Questions

4. *An insurer has different premiums for male and female customers. Its experience for accident year 2016 is given below. There was a rate change on 1st October 2015, which affects some policies in 2016.*

Sex	*Differential before rate change*	*Current differential*	*Earned premiums*	*Loss payments*
Male	*1*	*1*	*7,300*	*6,100*
Female	*0.81*	*0.77*	*5,600*	*4,300*

Before the rate change, the base premium was $1,250. The current base premium is $1,320. Assuming that policies are sold uniformly over the year, calculate the new premimums for policy year 2018 assuming 3% annual inflation and a permissible loss ratio of 0.80.

Assuming policies were issued uniformly over time, at time t in 2016 (so t is the time elapsed since 1st January 2016) the proportion of in-force policies that were issued before the rate change is $1-t-\frac{3}{12}$. The average proportion of policies in-force from before the rate change throughout 2016 is therefore $\int_0^{\frac{9}{12}}\left(1-t-\frac{3}{12}\right)\,dt=\frac{1}{2}\left(\frac{9}{12}\right)^2=\frac{9}{32}=0.28125$

We now need to adjust the earned premiums for 2016 to use the current rates. For male policyholders, we have that 71.9% of policies were at the current rate of $1,320, while 28.1% of policies were at the old premium of $1,250. The increase factor in earned premiums from changing the premiums from the old rate to the new rate is therefore $\frac{1320}{1320\times\frac{23}{32}+1250\times\frac{9}{32}}=1.015141$. The earned premiums for male policyholders at current rates is therefore $7300\times 1.015141=7410.5293$. For female policyholders the current premium is $0.77\times 1320=\$1,016.40$, while the former premium was $0.81\times 1250=\$1,012.50$. The adjustment needed for earned premiums is therefore $\frac{1016.40}{1016.40\times\frac{23}{32}+1012.50\times\frac{9}{32}}=1.00108034239$ The earned premiums at the current rate is therefore $5600\times 1.00108034239=5606.04991738$. The adjusted loss ratios for male and female policyholders in 2016 were therefore $\frac{6100}{7410.5293}=0.823153077608$ and $\frac{4300}{5606.04991738}=0.767028489466$ respectively. The current differential therefore needs to be multiplied by $\frac{0.767028489466}{0.823153077608}=0.931817556578$, so the new differential is $0.77\times 0.931817556578=0.717499518565$. The loss ratio for the base premium is 0.823153077608, so the premium needs to be increased by a factor $\frac{0.823153077608}{0.80}=1.02894134701$. Furthermore, applying inflation, we need to multiply by a factor $\frac{1.09280656402}{1.01492610407}$ (see Question 2), so the new base premium is $1320\times 1.02894134701\times\frac{1.09280656402}{1.01492610407}=\$1462.42$, and the premium for female policyholders is $1462.42439387\times 0.717499518565=\$1049.29$.

5. *An insurer classifies tenant's insurance policyholders into single or family, and into apartment or house. It has the following data from policy year 2016:*

Number of policies *loss payments*

	apartment	*house*
Single	*437*	*32*
Family	*128*	*204*

	apartment	*house*
Single	*$72,400*	*$9,800*
Family	*$42,600*	*$69,000*

(a) If the base classes are single and apartment, the base rate is $210, and the differentials are 1.44 for family and 1.25 for house, calculate the new premiums which give an expense ratio of 0.2 using the loss-ratio method.

Multiplying the number of policies by the premium gives the annual earned premiums

	apartment	house
Single	$437\times 210=91,770$	$32\times 210\times 1.25=8,400$
Family	$128\times 210\times 1.44=38,707.2$	$204\times 210\times 1.44\times 1.25=77,112$

The loss ratios for single and family are therefore $\frac{82200}{100170}=0.820604971548$ and $\frac{111600}{115819.2}=0.963570806913$ respectively, so the new differential for family is $1.44\times\frac{0.963570806913}{0.820604971548}=1.69087686532$. The loss ratios for apartments and houses are $\frac{115000}{130477.2}=0.881380041877$ and $\frac{78800}{85512}=0.92150809243$. The new differential for houses is therefore $\frac{0.92150809243}{0.881380041877}\times 1.25=1.30691082259$. Using these differentials to balance back, with these differentials at the current base premium, we get total earned premiums of

$$210\left(437+32\times 1.3069108+128\times 1.6908769+204\times 1.6908769\times 1.3069108\right)=240672.125646$$

and the loss ratio would be $\frac{193800}{240672.125646}=0.805244892735$. The base premium for 2016 therefore needs to be adjusted by a factor $\frac{0.805244892735}{0.80}=1.00655611592$. So the new base premium is $210\times 1.00655611592=\$211.38$. The premium for single house tennants is $211.376784343\times 1.30691082259=\$276.25$. The premium for family apartment tennants is $211.376784343\times 1.69087686532=\$357.41$. The premium for family house tennants is $211.376784343\times 1.69087686532\times 1.30691082259=\$467.11$.

(b) Repeat part (a) based on differentials of 0.85 for family and 0.95 for house.

Multiplying the number of policies by the premium gives the annual earned premiums

	apartment	house
Single	$437\times 210=91,770$	$32\times 210\times 0.95=6,384$
Family	$128\times 210\times 0.85=22,848$	$204\times 210\times 0.85\times 0.95=34,593.3$

The loss ratios for single and family are therefore $\frac{82200}{98154}=0.837459502415$ and $\frac{111600}{57441.3}=1.94285296468$ respectively, so the new differential for family is $0.85\times\frac{1.94285296468}{0.837459502415}=1.97194612422$. The loss ratios for apartments and houses are $\frac{115000}{114618}=1.00333280986$ and $\frac{78800}{40977.3}=1.92301591369$. The new differential for houses is therefore $\frac{1.92301591369}{1.00333280986}\times 0.95=1.82079674865$. Using these differentials to balance back, with these differentials at the current base premium, we get total earned premiums of

$$210\left(437+32\times 1.8207967+128\times 1.9719461+204\times 1.9719461\times 1.8207967\right)=310829.246809$$

and the loss ratio would be $\frac{193800}{310829.246809} = 0.623493451757$. The base premium for 2016 therefore needs to be adjusted by a factor $\frac{0.623493451757}{0.80} = 0.779366814696$. The new base premium is $210 \times 0.779366814696 = \$163.67$. The premium for single house tennants is $163.667031086 \times 1.82079674865 = \$298.00$. The premium for family apartment tennants is $163.667031086 \times 1.97194612422 = \$322.74$. The premium for family house tennants is $163.667031086 \times 1.97194612422 \times 1.82079674865 = \$587.65$.