ACSC/STAT 4703, Actuarial Models II
Fall 2018
Toby Kenney
Sample Final Examination
Model Solutions

This Sample examination has more questions than the actual final, in order to cover a wider range of questions. Estimated times are provided after each question to help your preparation.

1. *An insurance company sells home insurance. It estimates that the standard deviation of the aggregate annual claim is \$5,326 and the mean is \$1,804.*

   (a) How many years history are needed for an individual or group to be assigned full credibility? (Use $r = 0.05$, $p = 0.95$.) [5 mins.]

   The variance of the mean of a sample of n observations from an individual is $\frac{5326^2}{n}$, so a 95% confidence interval for this individual is their mean plus or minus $1.96 \times \frac{5326}{\sqrt{n}}$. We want the relative error in this estimate to be at most 5%. That is we want

$$1.96 \times \frac{5326}{\sqrt{n}} = 0.05 \times 1804$$
$$n = \left(\frac{10438.96}{90.2}\right)^2 = 13393.73$$

   (b) What is the Credibility premium, using limited fluctuation credibility, for an individual who has claimed a total of \$42,381 in the past 19 years? [5 mins.]

   This individual's average annual aggregate claims are $\frac{42381}{19} = \$2230.58$. The credibility is $\sqrt{\frac{19}{13393.73}} = 0.03766396$, so the credibility premium is $0.03766396 \times 2230.58 + 0.96233604 \times 1804 = \$1820.07$.

2. *For a car insurance policy, the book premium for claim severity is \$2,300. An individual has made 7 claims in the past 12 years, with average claim severity \$1,074. Calculate the credibility estimate for claim severity for this individual using limited fluctuation credibility, if the standard for full credibility is:*

   (a) 157 claims. [5 mins.]

   If the standard for full credibility is 157 claims, then this individual's credibility is $\sqrt{\frac{7}{157}} = 0.2111539$, and the credibility estimate is $0.2111539 \times 1074 + 0.7888461 \times 2300 = 2041.13$.

   (b) 284 years. [5 mins.]

   If the standard for full credibility is 157 claims, then this individual's credibility is $\sqrt{\frac{12}{284}} = 0.2055566$, and the credibility estimate is $0.2055566 \times 1074 + 0.7944434 \times 2300 = 2047.99$.

3. *A worker's compensation insurance company classifies workplaces as "safe" or "hazardous". Claims from hazardous workplaces follow a Gamma distribution with $\alpha = 0.1021749$, $\theta = 1066798$ (mean \$109,000 and standard deviation*

\$341,000). Claims from safe workplaces follow a Gamma distribution with $\alpha = 0.01209244$, $\theta = 2646281$ (mean \$32,000 and standard deviation \$261,000). 94% of workplaces are classified as safe.

[You may need the following values:

$$\Gamma(0.01209244) = 82.13091$$
$$\Gamma(0.1021749) = 9.302457$$

]

(a) Calculate the expectation and variance of claim size for a claim from a randomly chosen workplace. [5 mins.]

The expectation is $0.94 \times 32000 + 0.06 \times 109000 = \$36,620$. The variance is $(109000 - 32000)^2 \times 0.94 \times 0.06 + 0.94 \times 261000^2 + 0.06 \times 341000^2 = 71,345,000,000$.

(b) The last 2 claims from a particular workplace are \$488,200 and \$17,400. Calculate the expectation and variance for the next claim size from this workplace. [10 mins.]

If the workplace is safe, the likelihood of these claim sizes is

$$\left(\frac{488200^{-0.98790756}e^{-\frac{488200}{2646281}}}{2646281^{0.01209244}\Gamma(0.01209244)}\right)\left(\frac{17400^{-0.98790756}e^{-\frac{17400}{2646281}}}{2646281^{0.01209244}\Gamma(0.01209244)}\right) = 1.32923 \times 10^{-14}$$

If the workplace is hazardous, the likelihood of these claim sizes is

$$\left(\frac{488200^{-0.8978251}e^{-\frac{488200}{1066798}}}{1066798^{0.1021749}\Gamma(0.1021749)}\right)\left(\frac{17400^{-0.8978251}e^{-\frac{17400}{1066798}}}{1066798^{0.1021749}\Gamma(0.1021749)}\right) = 5.134517 \times 10^{-13}$$

The posterior probability that the workplace is safe is therefore $\frac{0.94 \times 1.32923 \times 10^{-14}}{0.94 \times 1.32923 \times 10^{-14} + 0.06 \times 5.134517 \times 10^{-13}} = 0.2885502$, so the expectation is $0.2885502 \times 32000 + 0.7114498 \times 109000 = \$86,781.63$.

The variance is $77000^2 \times 0.2885502 \times 0.7114498 + 0.2885502 \times 261000^2 + 0.7114498 \times 341000^2 = 103,601,580,743$.

4. *An insurance company sets the book pure premium for its home insurance at \$791. The expected process variance is 6,362,000 and the variance of hypothetical means is 341,200. If an individual has no claims over the last 8 years, calculate the credibility premium for this individual's next year's insurance using the Bühlmann model. [5 mins.]*

   The credibility is $Z = \frac{8}{8 + \frac{6362000}{341200}} = 0.3002332$. Therefore the premium is $0.6997668 \times 791 = \$553.52$.

5. *An insurance company is reviewing the premium for an individual with the following past claim history:*

Year	*1*	*2*	*3*	*4*	*5*
Exposure	*0.2*	*1*	*1*	*0.4*	*0.8*
Aggregate claims	*0*	*\$2,592*	*0*	*\$147*	*\$1,320*

The usual premium per unit of exposure is \$2,700. The expected process variance is 123045 and the variance of hypothetical means is 36403 (both per unit of exposure). Calculate the credibility premium for this individual if she has 0.6 units of exposure in year 6. [10 mins.]

The credibility of the policyholder's experience is $\frac{3.4}{3.4 + \frac{123045}{36403}} = 0.5014691$. The policyholder's aggregate claims were \$4,059, so average claims per unit of exposure are $\frac{4059}{3.4} = \$1,193.53$. The credibility premium per unit of exposure is therefore $0.5014691 \times 1193.53 + 0.4985309 \times 2700 = \$1,944.70$. This is for a whole unit of exposure. Since the policyholder has 0.6 units of exposure, the credibility premium is $0.6 \times 1944.70 = \$1,166.82$.

6. *An insurance company has 3 years of past history on a homeowner, denoted X_1, X_2, X_3. Because the individual moved house at the end of the second year, the third year has a different mean and variance, and is not as correlated with the other two years. It has the following*

$$\begin{aligned}
\mathbb{E}(X_1) &= 1,322 & \operatorname{Var}(X_1) &= 226,000\\
\mathbb{E}(X_2) &= 1,322 & \operatorname{Var}(X_2) &= 226,000\\
\mathbb{E}(X_3) &= 4,081 & \operatorname{Var}(X_3) &= 1,108,000\\
\mathbb{E}(X_4) &= 4,081 & \operatorname{Var}(X_4) &= 1,108,000\\
\operatorname{Cov}(X_1,X_2) &= 214 & \operatorname{Cov}(X_1,X_3) &= 181\\
\operatorname{Cov}(X_2,X_3) &= 181 & \operatorname{Cov}(X_1,X_4) &= 181\\
\operatorname{Cov}(X_2,X_4) &= 181 & \operatorname{Cov}(X_3,X_4) &= 861
\end{aligned}$$

It uses a formula $\hat{X}_4 = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3$ to calculate the credibility premium in the fourth year. Calculate the values of α_0, α_1, α_2 and α_3. [15 mins.]

The company needs to choose α_0, α_1, α_2 and α_3 to satisfy:

$$\begin{aligned}
\mathbb{E}(X_4) &= \alpha_0 + \alpha_1\mathbb{E}(X_1) + \alpha_2\mathbb{E}(X_2) + \alpha_3\mathbb{E}(X_3)\\
\operatorname{Cov}(X_4,X_1) &= \alpha_1 \operatorname{Var}(X_1) + \alpha_2 \operatorname{Cov}(X_2,X_1) + \alpha_3 \operatorname{Cov}(X_3,X_1)\\
\operatorname{Cov}(X_4,X_2) &= \alpha_1 \operatorname{Cov}(X_1,X_2) + \alpha_2 \operatorname{Var}(X_2) + \alpha_3 \operatorname{Cov}(X_3,X_2)\\
\operatorname{Cov}(X_4,X_1) &= \alpha_1 \operatorname{Cov}(X_1,X_3) + \alpha_2 \operatorname{Cov}(X_2,X_3) + \alpha_3 \operatorname{Var}(X_3)
\end{aligned}$$

Substituting the values gives:

$$\begin{aligned}
4081 &= \alpha_0 + 1322\alpha_1 + 1322\alpha_2 + 4081\alpha_3\\
181 &= 226000\alpha_1 + 214\alpha_2 + 181\alpha_3\\
181 &= 214\alpha_1 + 226000\alpha_2 + 181\alpha_3\\
861 &= 181\alpha_1 + 181\alpha_2 + 1108000\alpha_3
\end{aligned}$$

By symmetry, we see that α_1 and α_2 are equal. This gives

$$
\begin{aligned}
181 &= 226214\alpha_1 + 181\alpha_3 \\
861 &= 362\alpha_1 + 1108000\alpha_3 \\
226214 \times 861 - 362 \times 181 &= (226214 \times 1108000 + 362 \times 181)\alpha_3 \\
\alpha_3 &= \frac{194704732}{250,645,046,478} = 0.0007768146 \\
\alpha_1 &= \frac{181 - 181 \times 0.0007768146}{226214} = 0.0007995058 \\
\alpha_0 &= 4081 - 1322 \times 2 \times 0.0007995058 - 4081 \times 0.0007768146 = 4075.716
\end{aligned}
$$

The values are:

$$
\begin{aligned}
\alpha_0 &= 4075.716 \\
\alpha_1 &= 0.0007995058 \\
\alpha_2 &= 0.0007995058 \\
\alpha_3 &= 0.0007768146
\end{aligned}
$$

7. *An insurance company has the following previous data on aggregate claims:*

Policyholder	*Year 1*	*Year 2*	*Year 3*	*Year 4*	*Mean*	*Variance*
1	*1,210*	*246*	*459*	*1,461*	*944.00*	*340158.00*
2	*0*	*0*	*0*	*0*	*0.00*	*0.00*
3	*0*	*2,185*	*0*	*0*	*548.25*	*1202312.25*
4	*809*	*0*	*0*	*1,725*	*633.50*	*674939.00*
5	*0*	*0*	*0*	*0*	*0.00*	*0.00*

Calculate the Bühlmann credibility premium for policyholder 3 in Year 5. [15 mins.]

The expected process variance is $\frac{1}{5}(340158 + 0 + 1202312.25 + 674939 + 0) = 443421.85$. The population mean is $\frac{944+0+548.25+633.50+0}{5} = 405.15$.

total variance of estimated means is $\frac{(944-405.15)^2+(-405.15)^2+(548.25-405.15)^2+(633.50-405.15)^2+(-405.15)^2}{4} = 172318.425$. The variance of hypothetical means is therefore $172318.425 - \frac{443421.85}{4} = 61462.96$. The credibility of 4 years of experience is therefore $\frac{4}{4+\frac{443421.85}{61462.96}} = 0.3566825$. The premium for policyholder 3 is therefore $0.3566825 \times 548.25 + 0.6433175 \times 405.15 = \$456.19$.

8. *An insurance company collects the following claim frequency data for 7,000 customers insured for the past 3 years:*

No. of claims	*Frequency*
0	*1,494*
1	*2,460*
2	*1,810*
3	*827*
4	*302*
5	*72*
6	*31*
7	*3*
8	*1*
>8	*0*

It assumes that the number of claims an individual makes in a year follows a Poisson distribution with parameter Λ*, which may vary between individuals.*

Find the credibility estimate for the expected number of claims per year for an individual who has made 4 claims in the past 3 years. [15 mins.]

The total number of claims in the past 3 years was $1\times2460+2\times1810+3\times827+4\times302+5\times72+6\times31+7\times3+8\times1=10,344$. The total number of policyholders is $1491+2461+1810+831+302+72+30+2+1=7,000$. The average number of claims per policyholder per year is therefore $\frac{10344}{21000}=0.492571428571$. This is also the expected process variance. The variance of estimated means is

$$\frac{1}{6999}\left(1493\times0.492571428571^2+2460\left(\frac{1}{3}-0.492571428571\right)^2+1810\left(\frac{2}{3}-0.492571428571\right)^2+827(1-0.492571428571)+\right.$$

$$+31(2-0.492571428571)$$

The variance due to the Poisson sampling is $\frac{0.492571428571}{3}=0.16419047619$. Therefore, the variance of hypothetical means is $0.185843829802-0.16419047619=0.021653353612$. The credibility of 3 year's experience is $\frac{3}{3+\frac{0.492571428571}{0.021653353612}}=0.116513707423$. The expected number of claims is therefore $0.116513707423\times\frac{4}{3}+0.883486292577\times0.492571428571=0.590531715155$.

9. *An insurance company starts a new line of insurance in 2016, and collects a total of $1,900,000 in premiums that year, and the estimated incurred losses for accident year 2016 are $1,384,000. Half of the premium payments are made at the beginning of the year, and the other half are uniformly distributed over the year. An actuary is using this data to estimate rates for premium year 2018. Claims are subject to 4% inflation per year. By what percentage should premiums increase from 2016 in order to achieve a loss ratio of 0.75? [15 mins]*

The earned premiums for 2016 are $1900000\left(\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\right)=\$1,475,000$. [Half the policies earn the full premium in 2016, while the other half earn on average half of the premium collected in 2016.] This means that the loss ratio is $\frac{1384000}{1425000}=0.97122807018$. For losses in accident year 2016, the average inflation from the start of 2016 to the time of loss is given by $\int_0^1\frac{2}{3}(t+1)(1.04)^t\,dt$. [The proportion of policies in force at time t during 2016 is proporional to

$t+1$. The $\frac{2}{3}$ is a normalisation constant to get the pdf of the time of a random loss.] We calculate

$$\begin{aligned}\int_0^1 \frac{2}{3}(t+1)(1.04)^t\,dt &= \frac{2}{3}\int_0^1 (t+1)e^{\log(1.04)t}\,dt\\ &= \frac{2}{3}\left(\int_0^1 te^{\log(1.04)t}\,dt + \left[\frac{e^{\log(1.04)t}}{\log(1.04)}\right]_0^1\right)\\ &= \frac{2}{3}\left(\left[\frac{te^{\log(1.04)t}}{\log(1.04)}\right]_0^1 - \int_0^1 \frac{e^{\log(1.04)t}}{\log(1.04)}\,dt + \frac{0.04}{\log(1.04)}\right)\\ &= \frac{2}{3}\left(\frac{1.04}{\log(1.04)} - \left[\frac{e^{\log(1.04)t}}{\log(1.04)^2}\right]_0^1 + \frac{0.04}{\log(1.04)}\right)\\ &= \frac{2}{3}\left(\frac{1.04}{\log(1.04)} - \frac{0.04}{\log(1.04)^2} + \frac{0.04}{\log(1.04)}\right)\\ &= 1.02209143289\end{aligned}$$

For policy year 2018, the average inflation from the start of the year to the time of loss is

$$\begin{aligned}\int_0^1 te^{\log(1.04)t}\,dt + 1.04\int_0^1 (1-t)e^{\log(1.04)t}\,dt &= 1.04\int_0^1 e^{\log(1.04)t}\,dt - 0.04\int_0^1 te^{\log(1.04)t}\,dt\\ &= 1.04\times\frac{0.04}{\log(1.04)} - 0.04\left(\frac{1.04}{\log(1.04)} - \frac{0.04}{\log(1.04)^2}\right)\\ &= 1.04013332308\end{aligned}$$

The inflation from accident year 2016 to policy year 2018 is therefore $(1.04)^2 \times \frac{1.04013332308}{1.02209143289} = 1.10069233147$. The increase in premium is therefore $\frac{0.97122807018}{0.75} \times 1.10069233147 = 1.42536438527$, so a 42.54% increase in premium is required.

10. *An insurer classifies policies into three classes — single, couple and family. The experience from policy year 2016 is:*

Age Class	*Current differential*	*Earned premiums*	*Loss payments*
Single	*0.74*	*4,740*	*3,940*
Couple	*0.93*	*4,490*	*3,880*
Family	*1*	*5,670*	*4,930*

The base premium was \$420. Claim amounts are subject to 4% annual inflation. If the expense ratio is 25%, calculate the new premiums for each age class for policy year 2018. [15 mins]

Using the loss ratio method, the loss ratios are:

Class	loss ratio
Single	$\frac{3940}{4740} = 0.831223628692$
Couple	$\frac{3880}{4490} = 0.864142538976$
Family	$\frac{4930}{5670} = 0.869488536155$

The new differentials for couples should therefore be $0.93 \times \frac{0.864142538976}{0.869488536155} = 0.92428195178$. The new differential for singles should be $0.74 \times \frac{0.831223628692}{0.869488536155} = 0.70743369194$. Using these differentials, the total earned premiums in policy year 2016 would have been $5670 + 4490 \times \frac{0.92428195178}{0.93} + 4740 \times \frac{0.70743369194}{0.74} = 14663.7931034$, so the overall loss ratio would have been $\frac{12750}{14663.7931034} = 0.869488536158$. The target loss ratio is $1 - 0.25 = 0.75$, so the increase in base premium before inflation is $\frac{0.869488536158}{0.75} = 1.15931804821$. Two years of inflation is $(1.04)^2$, so the increase in base premium is $1.15931804821 \times (1.04)^2 = 1.25391840094$. The new base premium is $420 \times 1.25391840094 =$ \$526.645728395, and the new premium for a couple is $526.645728395 \times 0.92428195178 =$ \$486.77 and the new premium for a single policyholder is $526.645728395 \times 0.70743369194 =$ \$372.57.

11. *An insurer has different premiums for personal and commercial vehicles. Its experience for accident year 2016 is given below. There was a rate change on 1st August 2015, which affects some policies in 2016.*

Type	*Differential before rate change*	*Current differential*	*Earned premiums*	*Loss payments*
Personal	*1*	*1*	*11,300*	*9,800*
Commercial	*1.51*	*1.67*	*7,600*	*6,300*

Before the rate change, the base premium was \$950. The current base premium is \$1,020. Assuming that policies were sold uniformly over the year, calculate the new premimums for policy year 2018 assuming 6% annual inflation and a permissible loss ratio of 0.75. [15 mins]

The old premium applied for $\frac{7}{12}$ of 2015. Policies with this premium were therefore in force for $\frac{1}{2}\left(\frac{7}{12}\right)^2 = \frac{49}{288}$ of earned premium in 2016. Adjusting to the new premiums, the earned premium for personal in 2016 is $11300 \times \frac{1020}{1020\times\frac{239}{288}+950\times\frac{49}{288}} = 11433.4998106$. The adjusted earned premium for commercial policies in 2016 is $7600 \times \frac{1.67\times1020}{1.67\times1020\times\frac{239}{288}+1.51\times950\times\frac{49}{288}} = 7809.75640923$.

This means that the adjusted loss ratios are $\frac{9800}{11433.4998106} = 0.85713037673$ and $\frac{6300}{7809.75640923} = 0.80668329073$. The differential needs to be adjusted by a factor of $\frac{0.80668329073}{0.85713037673}$, so the new differential is $1.67 \times \frac{0.80668329073}{0.85713037673} = 1.57171082964$. Using this differential, total adjusted earned premiums in 2016 would be $11433.4998106+7809.75640923\times \frac{0.80668329073}{0.85713037673} = 18783.6068317$. The loss ratio is then $\frac{16100}{18783.6068317} = 0.85713037673$. The target loss ratio is 0.75, so without inflation, premiums need to be increased by a factor $\frac{0.85713037673}{0.75} = 1.14284050231$. Losses in accident year 2016 experience average inflation $\int_0^1 e^{\log(1.06)t}\,dt = \frac{0.06}{\log(1.06)} = 1.02970867194$ from the start of the year, while losses in policy year 2018 experience average inflation

$$\begin{aligned}
\int_0^1 te^{\log(1.06)t}\,dt + 1.06\int_0^1 (1-t)e^{\log(1.06)t}\,dt &= 1.06\int_0^1 e^{\log(1.06)t}\,dt - 0.06\int_0^1 te^{\log(1.06)t}\,dt \\
&= 1.06 \times \frac{0.06}{\log(1.06)} - 0.06\left(\frac{1.06}{\log(1.06)} - \frac{0.06}{\log(1.06)^2}\right) \\
&= 1.06029994908
\end{aligned}$$

from the start of 2018. The base premium therefore needs to change by a factor $1.14284050231 \times (1.06)^2 \times \frac{1.06029994908}{1.02970867194} = 1.32224436298$. The new base premium is $1.32224436298 \times 1020 = \$1,348.69$, and the new premium for commercial policies is $1348.68925024 \times 1.57171082964 = \$2,119.75$.

12. *An insurance company has the following data for accident year 2017:*

		Earned Premiums		*Loss Payments*	
		House	*Appartment*	*House*	*Appartment*
Differential		*1*	*0.88*	*1*	*0.88*
Halifax	*1*	*5,200*	*4,100*	*4,150*	*3,600*
Dartmouth	*0.84*	*3,700*	*2,900*	*2,080*	*2,430*
Bedford	*1.25*	*4,400*	*2,500*	*3,820*	*2,030*

The base premium in 2017 was \$840. Calculate new premiums for policy year 2018 using inflation of 3% per year and expense ratio of 0.2.

We first calculate the new differentials. We obtain the following loss ratios:

Halifax	$\frac{7750}{9300} = 0.833333333333$
Dartmouth	$\frac{4510}{6600} = 0.683333333333$
Bedford	$\frac{5850}{6900} = 0.847826086957$
House	$\frac{10050}{13300} = 0.755639097744$
Apartment	$\frac{8060}{9500} = 0.848421052632$

The new differentials are therefore

Dartmouth	$0.84 \times \frac{0.683333333333}{0.833333333333} = 0.6888$
Bedford	$1.25 \times \frac{0.847826086957}{0.833333333333} = 1.27173913044$
Apartment	$0.88 \times \frac{0.848421052632}{0.755639097744} = 0.988051741292$

Balancing back the adjusted earned premiums are

$$5200+4100\times\frac{0.988051741292}{0.88}+3700\times\frac{0.6888}{0.84}+2900\times\frac{0.6888}{0.84}\times\frac{0.988051741292}{0.88}+4400\times\frac{1.27173913044}{1.25}+2500\times\frac{1.27173913044}{1.25}\times$$

The loss ratio is therefore $\frac{18110}{22839.7118581} = 0.792917183567$. To obtain an expense ratio of 0.2, the base premium therefore needs to be multiplied by $\frac{0.792917183567}{0.8} = 0.991146479459$.

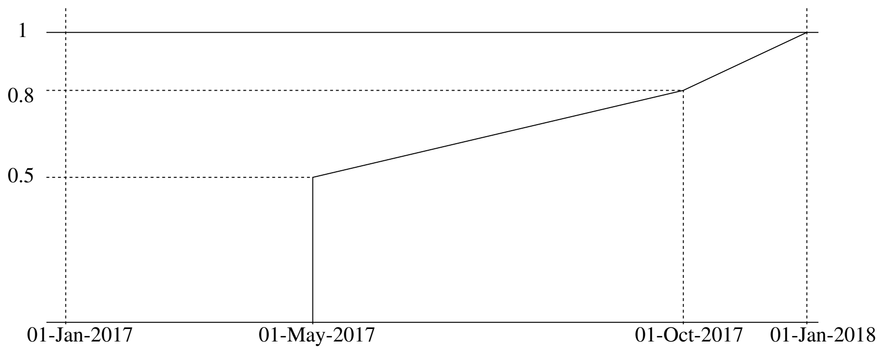
The expected inflation from the start of 2017 to a random loss is $\int_0^1 (1.03)^t\,dt = \frac{0.03}{\log(1.03)} = 1.01492610407$. The expected inflation from the start of 2018 to a random loss in policy year 2018 is

$$\begin{aligned}
\int_0^1 t(1.03)^t\,dt + \int_1^2 (2-t)(1.03)^t\,dt &= \int_0^1 t(1.03)^t\,dt + 1.03\int_0^1 (1-t)(1.03)^t\,dt \\
&= 1.03\int_0^1 (1.03)^t\,dt - 0.03\int_0^1 t(1.03)^t\,dt \\
&= 1.03\frac{0.03}{\log(1.03)} - 0.03\left(\frac{1.03}{\log(1.03)} - \frac{0.03}{\log(1.03)^2}\right) \\
&= \left(\frac{0.03}{\log(1.03)}\right)^2 \\
&= 1.03007499672
\end{aligned}$$

The new base premium is therefore $840 \times 0.991146479459 \times \frac{1.03\times 1.03007499672}{1.01492610407} = 870.339664328$. The new premiums are therefore:

	House	Appartment
Halifax	\$870.34	\$859.94
Dartmouth	\$599.49	\$592.33
Bedford	\$1,106.85	\$1,093.62

13. *An insurance company is calculating the premium for a new line of insurance it started in 2018. The new line of insurance started on 1st May 2018, and half of the policies started at that time. Due to an advertising campaign, the rate of policy purchases in November and December was twice the rate for the months from May to October. The annual premium in 2018 was \$600. The total premiums collected in 2018 were \$1,200,000 and the total losses were \$462,000. Assuming losses are uniformly distributed throughout the year, annual inflation is 5%, and the expense ratio is 0.2, calculate the new premium for policy year 2020.*



The number of policies in force at time t in the year 2018 is

$$f(t) = \begin{cases} 0 & \text{if } t < \frac{4}{12} \\ 0.6t + 0.3 & \text{if } \frac{4}{12} < t < \frac{10}{12} \\ 1.2t - 0.2 & \text{if } \frac{10}{12} < t < 1 \end{cases}$$

The total earned premiums for accident year 2018 are $1200000 \times \left(\frac{1}{2} \times \frac{6}{12} \times \left(\frac{1}{2} + 0.8\right) + \frac{1}{2} \times \frac{2}{12} \times (0.8 + 1)\right) = 570000$
The loss ratio is therefore $\frac{462000}{570000} = 0.810526315789$, so before inflation the premium is should be adjusted by a factor of $\frac{0.810526315789}{0.8} = 1.01315789474$.
Inflation from the start of 2018 to the average accident time in 2018 is given by

$$\frac{\int_{\frac{4}{12}}^{\frac{10}{12}} (0.6t + 0.3)\,(1.05)^t\,dt + \int_{\frac{10}{12}}^{1} (1.2t - 0.2)\,(1.05)^t\,dt}{\left(\frac{1}{2} \times \frac{6}{12} \times \left(\frac{1}{2} + 0.8\right) + \frac{1}{2} \times \frac{2}{12} \times (0.8 + 1)\right)}$$

$$= \frac{0.3\int_{\frac{4}{12}}^{\frac{10}{12}} (1.05)^t\,dt + 0.6\int_{\frac{4}{12}}^{\frac{10}{12}} t(1.05)^t\,dt + 1.2\int_{\frac{10}{12}}^{1} t(1.05)^t\,dt - 0.2\int_{\frac{10}{12}}^{1} (1.05)^t\,dt}{0.475}$$

$$= \frac{0.3(1.05^{\frac{10}{12}} - 1.05^{\frac{4}{12}}) - 0.2(1.05 - 1.05^{\frac{10}{12}}) + 0.6\left(\frac{10}{12}(1.05)^{\frac{10}{12}} - \frac{4}{12}(1.05)^{\frac{4}{12}} - \frac{(1.05)^{\frac{10}{12}} - (1.05)^{\frac{4}{12}}}{\log(1.05)}\right) + 1.2\left(1.05 - \frac{10}{12}1.05^{\frac{10}{12}} - \frac{1-1}{\log}\right.}{0.475\log(1.05)}$$

$$= \frac{1.05 - 0.5(1.05)^{\frac{4}{12}} - \frac{1.2(1.05) - 0.6(1.05)^{\frac{10}{12}} - 0.6(1.05)^{\frac{4}{12}}}{\log(1.05)}}{0.475\log(1.05)}$$

$$= 1.03492570259$$

Inflation from the start of 2020 to the average accident time in policy year 2020 is given by

$$\begin{aligned}
\int_0^1 t(1.05)^t\,dt + \int_1^2 (2-t)(1.05)^t\,dt &= \int_0^1 t(1.05)^t\,dt + (1.05)\int_0^1 (1-t)(1.05)^t\,dt \\
&= 1.05\int_0^1 (1.05)^t\,dt - 0.05\int_0^1 t(1.05)^t\,dt \\
&= \frac{1.05\times 0.05}{\log(1.05)} - 0.05\left(\frac{1.05}{\log(1.05)} - \frac{0.05}{\log(1.05)^2}\right) \\
&= \left(\frac{0.05}{\log(1.05)}\right)^2 \\
&= 1.05020830855
\end{aligned}$$

The premium is therefore $600 \times 1.01315789474 \times 1.05^2 \times \frac{1.05020830855}{1.03492570259} = \$680.10$.

14. *An insurance company has the following data on its policies:*

Policy limit	*Losses Limited to*			
	20,000	*50,000*	*100,000*	*500,000*
20,000	*1,400,000*			
50,000	*7,540,000*	*8,010,000*		
100,000	*22,600,000*	*24,100,000*	*28,700,000*	
500,000	*5,900,000*	*6,220,000*	*6,650,000*	*6,920,000*

Use this data to calculate the ILF from \$20,000 to \$500,000 using

(a) The direct ILF estimate. [5 mins]

The direct ILF estimate is $\frac{6920000}{5900000} = 1.17288135593$.

(b) The incremental method. [5 mins]

Using the incremental method the ILFs are:

\$20,000–\$50,000	$\frac{8010000+24100000+6220000}{7540000+22600000+5900000} = 1.06354051054$
\$50,000–\$100,000	$\frac{28700000+6650000}{24100000+6220000} = 1.16589709763$
\$100,000–\$500,000	$\frac{6920000}{6650000} = 1.04060150376$

So the ILF is $1.06354051054 \times 1.16589709763 \times 1.04060150376 = 1.29032379813$.

15. *For a certain line of insurance, the loss amount per claim follows a Pareto distribution with $\alpha = 4$. If the policy has a deductible per loss set at 0.1θ and a policy limit set at 2θ, by how much will the expected payment per loss increase if there is inflation of 5%? [10 mins]*

The expected payment per loss before inflation is

$$\begin{aligned}
\int_{0.1\theta}^{2\theta} S(x)\,dx &= \int_{0.1\theta}^{2\theta} \frac{\theta^4}{(\theta+x)^4}\,dx \\
&= \theta^4 \int_{1.1\theta}^{3\theta} u^{-4}\,du \\
&= \theta^4 \left[-\frac{u^{-3}}{3}\right]_{1.1\theta}^{3\theta} \\
&= \theta^4 \left(\frac{(1.1\theta)^{-3}}{3} - \frac{(3\theta)^{-3}}{3}\right) \\
&= \frac{\theta}{3}\left((1.1)^{-3} - 3^{-3}\right) \\
&= 0.2380925879955\theta
\end{aligned}$$

After inflation, the expected payment per loss is

$$\begin{aligned}
\int_{0.1\theta}^{2\theta} S(x)\,dx &= \int_{0.1\theta}^{2\theta} \frac{(1.05\theta)^4}{(1.05\theta+x)^4}\,dx \\
&= 1.05^4\theta^4 \int_{1.15\theta}^{3.05\theta} u^{-4}\,du \\
&= \theta^4 \left[-\frac{u^{-3}}{3}\right]_{1.15\theta}^{3.05\theta} \\
&= 1.05^4\theta^4 \left(\frac{(1.15\theta)^{-3}}{3} - \frac{(3.05\theta)^{-3}}{3}\right) \\
&= \frac{1.05^4\theta}{3}\left((1.15)^{-3} - 3.05^{-3}\right) \\
&= 0.2521247598844\theta
\end{aligned}$$

The increase in expected payment per loss is $\frac{0.252124759844}{0.238092587955} - 1 = 5.893577792\%$.

16. *An insurance company charges a risk charge equal to the square of the average loss amount, divided by 100,000. It has the following data on a set of 1,200 claims from policies with limit $1,000,000.*

Losses Limited to	*50,000*	*100,000*	*500,000*	*1,000,000*
Total claimed	*16,700,000*	*20,880,000*	*27,030,000*	*32,410,000*

Calculate the ILF from $100,000 to $1,000,000. [10 mins]

For limit $100,000, the expected loss amount is $\frac{20880000}{1200} = 17400$, and the risk charge is $\frac{17400^2}{100000} = 3027.6$. The premium is therefore $17400 + 3027.6 = 20427.6$. For limit $1,000,000, the expected loss amount is $\frac{32410000}{1200} = 27008.3333333$, and the risk charge is $\frac{27008.3333333^2}{100000} = 7294.50069443$, so the premium is $27008.3333333+7294.50069443 = 34302.8340277$. The ILF is therefore $\frac{34302.8340277}{20427.6} = 1.6792395596$.

17. *An insurer calculates the ILF on the pure premium from $1,000,000 to $2,000,000 on a particular policy is 1.092. A reinsurer offers excess-of-loss reinsurance of $1,000,000 over $1,000,000 for a loading of 30%. The original insurer*

uses a loading of 20% on policies with limit \$1,000,000. If the insurer buys the excess-of-loss reinsurance, what is the loading on its premium for policies with a limit of \$2,000,000? [10 mins]

Let m be the expected loss on the policy with limit \$1,000,000. With a 20% loading, the insurer charges $1.2m$ for the insurance. The expected payment on the reinsurance is $1.092m - m = 0.092m$. With a loading of 30%, the cost of the reinsurance is $0.092m \times 1.3 = 0.1196m$, so the total cost with a limit of \$2,000,000 is $1.2m + 0.1196m = 1.3196m$, and the expected payment is $1.092m$, so the loading is $\frac{1.3196m}{1.092m} - 1 = 20.842490842\%$.