ACSC/STAT 4703, Actuarial Models II
Winter 2020
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Sample Final Examination

This Sample examination has more questions than the actual final, in order to cover a wider range of questions. Estimated times are provided after each question to help your preparation.

1. Automobile insurance company A estimates that the standard deviation of aggregate annual claims for an individual is \$3,579 and the mean is \$1,824.

   (a) How many years history are needed for an individual or group to be assigned full credibility? (Use $r = 0.02$, $p = 0.90$.) [5 mins.]

   (b) If an individual has claimed \$6,000 in the past 5 years, what credibility premium should they pay? [5 mins.]

   (c) Insurance company B uses $r = 0.10$ and $p = 0.95$ for setting its credibility premiums. The individual from (b) claims \$1,200 every year. She switches to company B, where she has 0 years of experience. How many years will it take until the total premiums she has paid is lower than if she had not switched.

2. A home insurance company classifies policyholders into "low risk" and "high risk". Annual claims from low risk policyholders follow a Pareto distribution with $\theta = 1000$ and $\alpha = 5$ and claims from high risk policyholders follow a gamma distribution with $\alpha = 3$ and $\theta = 300$. 40% of policyholders are high risk.

   (a) What is the Bayesian premium for a policyholder who has claimed \$500 in one year?

   (b) What is the largest Bayes premium a policyholder could have to pay in Year 2?

3. A policyholder starts a new auto insurance policy. In the first year, he pays the book premium of \$760, and his aggregate claims are \$850. His premium for the second year is \$774, while the book premium is still \$760. This premium is calculated using Buhlmann credibility. If he claims \$420 in the second year, what premium will he pay in the third year (assuming the book premium remains \$760).

4. A health insurance company is reviewing the premium for a group with the following past claim history:

Year	1	2	3	4	5
Exposure	352	532	235	364	403
Aggregate claims	\$35,633	\$42,014	\$26,852	\$63,154	\$33,706

   The book premium per unit of exposure is \$153.59. The expected process variance is 23145 and the variance of hypothetical means is 303 (both per unit of exposure).

   (a) Calculate the premium per unit of exposure in Year 6.

   (b) If the company has 490 units of exposure in Year 6, what aggregate claims in Year 6 would cause it to have the same premium per unit of exposure in Year 7?

5. An insurance company has 3 years of past history on a homeowner, denoted X_1, X_2, X_3. Because the individual moved house at the end of the second year, the third year has a different mean and variance, and is not as correlated with the other two years. It has the following

$$\mathbb{E}(X_1) = 1,322 \qquad \text{Var}(X_1) = \quad 226,000$$
$$\mathbb{E}(X_2) = 1,322 \qquad \text{Var}(X_2) = \quad 226,000$$
$$\mathbb{E}(X_3) = 4,081 \qquad \text{Var}(X_3) = 1,108,000$$
$$\mathbb{E}(X_4) = 4,081 \qquad \text{Var}(X_4) = 1,108,000$$
$$\text{Cov}(X_1, X_2) = 214 \qquad \text{Cov}(X_1, X_3) = 181$$
$$\text{Cov}(X_2, X_3) = 181 \qquad \text{Cov}(X_1, X_4) = 181$$
$$\text{Cov}(X_2, X_4) = 181 \qquad \text{Cov}(X_3, X_4) = 861$$

It uses a formula $\hat{X_4} = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3$ to calculate the credibility premium in the fourth year. Calculate the values of α_0, α_1, α_2 and α_3. [15 mins.]

6. An insurance company has the following previous data on aggregate claims:

Policyholder	Year 1	Year 2	Year 3	Year 4	Mean	Variance
1	1,210	246	459	1,461	944.00	340158.00
2	0	0	0	0	0.00	0.00
3	0	2,185	0	0	548.25	1202312.25
4	809	0	0	1,725	633.50	674939.00
5	0	0	0	0	0.00	0.00

Calculate the Bühlmann credibility premium for policyholder 3 in Year 5. [15 mins.]

7. An insurance company collects the following claim frequency data for 7,000 customers insured for the past 3 years:

No. of claims	Frequency
0	1,492
1	2,460
2	1,810
3	827
4	302
5	72
6	31
7	3
8	1
> 8	0

It assumes that the number of claims an individual makes in a year follows a Poisson distribution with parameter Λ, which may vary between individuals.

Find the credibility estimate for the expected number of claims per year for an individual who has made 4 claims in the past 3 years. [15 mins.]

8. An actuary is reviewing claim data from accident year 2019 for a particular line of insurance. The earned premium is \$3,520,320, and the aggregate claims are \$2,560,600. At the start of the year, there are 7,400 policies in force. After 4 months, there are only 4,200 policies in force. After 8 months, there are 7,600 policies in force, and at the end of the year, there are 8,500 policies in force. Assuming the number of policies in force is linear between these

data points, what should be the percentage change in the premium for policy year 2021, if inflation is 5% and the permissible loss ratio is 0.80? Assume that policies are sold uniformly during 2021.

9. An insurer classifies policies into three classes — low risk, medium risk, and high risk. The experience from policy year 2018 is:

Class	Current differential	Earned premiums	Loss payments
low risk	0.72	4,740	3,940
medium risk	1	4,490	3,880
high risk	1.68	5,670	4,930

The base premium was $420. Claim amounts are subject to 5% annual inflation. If the expense ratio is 30%:

(a) calculate the new premiums for each age class for policy year 2021. [15 mins]

(b) The insurance company wants to reduce to two policy classes, with low risk as the base class. It will assign some policyholders from the medium risk class into low risk, and some into high risk. Assume that the loss ratio for the medium-risk policyholders reclassified as low-risk and the loss ratio for the medium-risk policyholders reclassified as high-risk are the same. If the new base premium after combining these classes is $480, what should the differential for the high risk class be?

10. An insurer has different premiums for personal and commercial vehicles. Its experience for accident year 2018 is given below. There was a rate change on 1st August 2017, which affects some policies in 2018.

Type	Differential before rate change	Current differential	Earned premiums	Loss payments
Personal	1	1	11,300	9,800
Commercial	1.51	1.67	7,600	6,300

Before the rate change, the base premium was $950. The current base premium is $1,020.

(a) Assuming that policies were sold uniformly over the year, calculate the new premimums for policy year 2020 assuming 6% annual inflation and a permissible loss ratio of 0.75. [15 mins]

(b) Suppose that twice as many policies are sold in April, May, June, July, August, September and October as in other months. What is the new premium in this case. [This rate of sale applies to both the rate change in the data, and the policies sold in 2020.] [15 mins]

11. An insurance company has the following data for accident year 2017 when the base premium was $840:

		Earned Premiums		Loss Payments	
		House	Apartment	House	Apartment
Differential		1	0.88	1	0.88
Halifax	1	5,200	4,100	4,350	?,???
Dartmouth	0.84	3,700	2,900	3,020	2,230
Bedford	1.25	4,400	2,500	3,550	2,330

Unfortunately, some records have been lost. The base premium for policy year 2020 using this data, inflation of 3% per year and expense ratio of 0.2 was calculated at $935. What is the missing value in the table?

12. An insurance company is calculating the premium for a new line of insurance it started in 2018. The new line of insurance started on 1st May 2018, and half of the policies started at that time. Due to an advertising campaign, the rate of policy purchases in November and December was twice the rate for the months from May to October. The annual premium in 2018 was $600. The total premiums collected in 2018 were $1,200,000 and the total losses were

$462,000. Assuming losses are uniformly distributed throughout the year, annual inflation is 5%, and the expense ratio is 0.2, calculate the new premium for policy year 2020.

13. An insurance company has the following data on its policies:

Policy limit	Losses Limited to			
	20,000	50,000	100,000	500,000
20,000	1,400,000			
50,000	7,540,000	8,010,000		
100,000	22,600,000	24,100,000	28,700,000	
500,000	5,900,000	6,220,000	6,650,000	6,920,000

(a) Use this data to calculate the ILF from $20,000 to $500,000 using the incremental method. [5 mins]

(b) A reinsurance company uses the ILF calculated in (a) to calculate its premiums. The reinsurance company offers excess-of-loss reinsurance of $450,000 over $50,000 for a premium of $240, which includes a 20% loading. How many policies in the dataset above had a policy limit of at least $50,000? [10 mins]

14. For a certain line of insurance, the loss amount per loss is modelled as a Pareto distribution with $\alpha = 5$. The policy has a deductible per loss set at $5,000 and a policy limit set at $1,000,000. After inflation of 5%, the expected payment per loss increases by 4.6%. What was the mean loss amount before the inflation? [10 mins]

15. An insurance company charges a risk charge equal to the square of the average loss amount, divided by 100,000. It purchases excess-of-loss reinsurance of $500,000 over $500,000. The loading on this reinsurance is 25%. The difference between the insurance company's premiums for policies with limit $500,000 and policies with limit $1,000,000 is exactly the reinsurance premium. If the insurer sells 500 policies with limit $500,000 and 500 policies with limit $1,000,000, what is the expected aggregate payment on this portfolio?

16. An insurer calculates the ILF on the pure premium from $1,000,000 to $2,000,000 on a particular policy is 1.092. A reinsurer offers excess-of-loss reinsurance of $1,000,000 over $1,000,000 for a loading of 30%. The original insurer uses a loading of 20% on policies with limit $1,000,000. If the insurer buys the excess-of-loss reinsurance, what is the loading on its premium for policies with a limit of $2,000,000? [10 mins]