ACSC/STAT 4703, Actuarial Models II
Fall 2020
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Homework Sheet 5
Model Solutions

Basic Questions

1. *An insurance company sets the book pure premium for its fire insurance at \$488. The expected process variance is 92,063 and the variance of hypothetical means is 56,243. If a company has aggregate claims of \$23,400 on policies covering a total of 36 properties, calculate the credibility premium for this company's next year's insurance using the Bühlmann model.*

The Bühlmann credibility is

$$Z = \frac{n}{n + \frac{\text{EPV}}{\text{VHM}}} = \frac{36}{36 + \frac{92063}{56243}} = 0.95650863492$$

Therefore the credibility premium is

$$0.95650863492 \times \frac{23400}{36} + 0.04349136508 \times 488 = \$642.95$$

2. *An insurance company has the following data on a Workers' compensation insurance policy for a company.*

Year	*1*	*2*	*3*	*4*	*5*
Exposure	*356*	*402*	*550*	*526*	*572*
Aggregate claims	*\$250,201*	*\$293,114*	*\$477,136*	*\$482,150*	*\$499,300*

The book premium is \$960 per unit of exposure. The variance of hypothetical means per unit of exposure is 589,000. The expected process variance per unit of exposure is 18,323,900. Using a Bühlmann-Straub model, calculate the credibility premium for Year 6 if the company has 611 units of exposure.

The company has a total of \$2,001,901 from 2,406 units of exposure. The credibility of 2406 units of exposure is therefore

$$Z = \frac{2406}{2406 + \frac{18323900}{589000}} = 0.987234805005$$

The credibility premium is therefore

$$0.987234805005 \times \frac{2001901}{2406} + 0.012765194995 \times 960 = \$833.68$$

per unit of exposure. The premium for 611 units of exposure is therefore $833.68 \times 611 = \$509,378.48$.

3. *An insurance company has the following previous data on aggregate claims:*

Policyholder	*Year 1*	*Year 2*	*Year 3*	*Year 4*	*Year 5*	*Mean*	*Variance*
1	*0.00*	*0.00*	*2984.19*	*0.00*	*0.00*	*596.838*	*1781077.99122*
2	*1401.86*	*0.00*	*0.00*	*5422.18*	*3521.14*	*2069.036*	*5589781.11628*
3	*0.00*	*0.00*	*0.00*	*512.54*	*861.47*	*274.802*	*156811.77912*
4	*0.00*	*597.94*	*0.00*	*288.63*	*488.99*	*275.112*	*75379.41947*

Calculate the Bühlmann credibility premium for each policyholder in Year 6.

The expected process variance is $\frac{1}{4}(1781077.99122+5589781.11628+156811.77912+75379.41947) = 1900762.57652$. The mean aggregate claim is $\frac{1}{4}(596.838+2069.036+274.802+275.112) = 803.947$. The variance of observed means is $\frac{(596.838-803.947)^2+(2069.036-803.947)^2+(274.802-803.947)^2+(275.112-803.947)^2}{3} = 734335.06802$. Of this, $\frac{1900762.57652}{5} = 380152.515304$ is due to process variance, so the variance of hypothetical means is $734335.06802 - 380152.515304 = 354182.552716$. This means that the credibility of 5 years of experience is

$$Z = \frac{5}{5 + \frac{1900762.57652}{354182.552716}} = 0.482317361842$$

The credibility premium for each policyholder in Year 6 is therefore given by

$$0.482317361842 \times 596.838 + 0.517682638158 \times 803.947 = \$704.05$$
$$0.482317361842 \times 2069.036 + 0.517682638158 \times 803.947 = \$1,414.12$$
$$0.482317361842 \times 274.802 + 0.517682638158 \times 803.947 = \$548.73$$
$$0.482317361842 \times 275.112 + 0.517682638158 \times 803.947 = \$548.88$$

4. *An insurance company observes the following numbers of claims from individuals over a seven-year period — that is, the following table gives the number of claims in the past seven years:*

No. of claims	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Frequency	*1,933*	*1,788*	*891*	*660*	*491*	*58*	*43*	*46*	*23*	*0*	*1*

Assuming the number of claims made by an individual in a year follows a Poisson distribution, calculate the credibility estimate for the expected claim frequency in the following year, of an individual who has made a total of 1 claim in the past 6 years. [Note that this is a different length of history from the individuals in the dataset.]

The number of policies in the data set is $1933 + 1788 + 891 + 660 + 491 + 58 + 43 + 46 + 23 + 0 + 1 = 5934$ The expected number of claims in a seven-year period is

$$\frac{1 \times 1788 + 2 \times 891 + 3 \times 660 + 4 \times 491 + 5 \times 58 + 6 \times 43 + 7 \times 46 + 8 \times 23 + 9 \times 0 + 10 \times 1}{5934} = \frac{8578}{5934} =$$

The expected square of the number of claims in a seven-year period is

$$\frac{1^2\times 1788+2^2\times 891+3^2\times 660+4^2\times 491+5^2\times 58+6^2\times 43+7^2\times 46+8^2\times 23+9^2\times 0+10^2\times 1}{5934} =$$

The variance of observed number of claims is therefore

$$\frac{5934}{5933}(4.3768115942-1.44556791372^2)=2.28753049654$$

For a Poisson distribution, the EPV is equal to the mean, so the VHM for a seven-year period is $2.28753049654-1.44556791372=0.84196258282$. Thus the credibility of 6 years of experience is

$$Z=\frac{\frac{6}{7}}{\frac{6}{7}+\frac{1.44556791372}{0.84196258282}}=0.332994426748$$

Therefore, the expected claim frequency for this policyholder in a one-year period is

$$0.332994426748\times\frac{1}{6}+0.667005573252\times\frac{1.44556791372}{7}=0.193242193263$$

Standard Questions

5. *Aggregate claims for a given individual policy are modelled as following a Pareto distribution with $\alpha=6$. The first 5 years of experience on this policy are:*

Policyholder	*Year 1*	*Year 2*	*Year 3*	*Year 4*	*Year 5*	*Mean*	*Variance*
1	*0.2*	*17.5*	*0.4*	*14.6*	*1.0*	*6.74*	*8.56551*
2	*1480.6*	*14.5*	*970.7*	*30.9*	*1873.1*	*873.96*	*840.39769*
3	*700.9*	*79.9*	*1417.4*	*2702.4*	*1.6*	*980.44*	*1118.40544*
4	*24.7*	*165.0*	*0.0*	*28.0*	*210.9*	*85.72*	*95.33927*

(a) Estimate the EPV and VHM.

The expected claim is $\frac{6.74+873.96+980.44+85.72}{4}=486.715$. A Pareto distribution with $\alpha=6$ and Θ a random variable has mean $\frac{\theta}{5}=0.2\Theta$ and variance $\frac{6\Theta^2}{5^2\times 4}=0.06\Theta^2$. Thus we have that the mean is $\mathbb{E}(\Theta)=\frac{486.715}{0.2}=2433.575$. The variance of hypothetical means is $\mathrm{Var}(0.2\Theta)=0.04\,\mathrm{Var}(\Theta)$, and the expected process variance is

$$0.06\mathbb{E}(\Theta^2)=0.06\left(\mathbb{E}(\Theta)^2+\mathrm{Var}(\Theta)\right)$$

The variance of observed means is

$$\mathrm{VHM}+\frac{\mathrm{EPV}}{5}=\frac{0.04\,\mathrm{Var}(\Theta)+0.06\times 2433.575^2+0.06\,\mathrm{Var}(\Theta)}{5}=0.02\,\mathrm{Var}(\Theta)+71067.4473676$$

From the data, the variance of observed means is

$$\frac{(6.74-486.715)^2+(873.96-486.715)^2+(980.44-486.715)^2+(85.72-486.715)^2}{3}=261632.018767$$

Thus, we get

$$\begin{aligned}0.02\,\mathrm{Var}(\Theta)+71067.4473676 &= 261632.018767\\ \mathrm{Var}(\Theta) &= \frac{261632.018767-71067.4473676}{0.02}\\ &= 9528228.56995\\ \mathrm{EPV} &= 0.06\left(2433.575^2+9528228.56995\right)\\ &= 927030.951036\\ \mathrm{VHM} &= 0.04\times 9528228.56995\\ &= 381129.142798\end{aligned}$$

(b) Calculate the credibility premium for policyholder 4 in the next year.

The credibility of 5 years of experience is

$$Z=\frac{5}{5+\frac{927030.951036}{381129.142798}}=0.672736757258$$

Therefore the credibility premium for policyholder 4 is

$$0.672736757258\times 85.72+0.327263242742\times 486.715=\$216.95$$

6. *Claim frequency in a year for an individual follows a Poisson with parameter Λt where Λ is the individual's risk factor and t is the individual's exposure in that year. An insurance company collects the following data:*

	Year 1		*Year 2*		*Year 3*	
Policyholder	*Exp*	*claims*	*Exp*	*claims*	*Exp*	*claims*
1	*454*	*5*	*531*	*7*	*450*	*3*
2	*617*	*1*	*616*	*2*	*539*	*0*
3	*728*	*5*	*651*	*2*	*804*	*3*
4	*767*	*2*	*761*	*4*	*832*	*3*

In Year 4, policyholder 3 has 793 units of exposure. Calculate the credibility estimate for claim frequency for policyholder 3.

The estimates for Λ_i from this data are given in the following table

Policyholder	Total Exp	Total claims	$\hat{\Lambda}$
1	1435	15	0.0104529616725
2	1772	3	0.00169300225734
3	2183	10	0.00458085203848
4	2360	9	0.00381355932203

Thus the overall mean is $\frac{0.0104529616725+0.00169300225734+0.00458085203848+0.00381355932203}{4} =$ 0.00513509382258 Under the Poisson model, this is also the EPV per unit of exposure. The variance of observed means is then $\frac{(0.0104529616725-0.00513509382258)^2+(0.00169300225734-0.00513509382258)^2+(0.00458085203848-0.00513509382258)^2+(0.00}{3}$ $1.4060450068 \times 10^{-5}$. However, the variance due to process variance is $0.00513509382258\left(\frac{1}{1435}+\frac{1}{1772}+\frac{1}{2183}+\frac{1}{2360}\right) = 1.10045687853 \times 10^{-5}$. Thus the variance of hypothetical means is $1.4060450068 \times 10^{-5} - 1.10045687853 \times 10^{-5} = 3.0558812827 \times 10^{-6}$. Thus, the credibility of n units of exposure is $\frac{n}{n+\frac{0.00513509382258}{3.0558812827\times 10^{-6}}}$. The credibility for the individuals in the data is

$$Z_1 = \frac{1435}{1435 + \frac{0.00513509382258}{3.0558812827\times 10^{-6}}} = 0.46061544885$$
$$Z_2 = \frac{1772}{1772 + \frac{0.00513509382258}{3.0558812827\times 10^{-6}}} = 0.51326657568$$
$$Z_3 = \frac{2183}{2183 + \frac{0.00513509382258}{3.0558812827\times 10^{-6}}} = 0.56504676705$$
$$Z_4 = \frac{2360}{2360 + \frac{0.00513509382258}{3.0558812827\times 10^{-6}}} = 0.58410101388$$

To ensure the estimates are balanced, we take a credibility-weighted mean for the book value.

$$\hat{\mu} = \frac{0.46061544885 \times 0.0104529616725 + 0.51326657568 \times 0.00169300225734 + 0.56504676705 \times 0.004580}{0.46061544885 + 0.51326657568 + 0.56504676705 + 0.58410}$$

Thus the credibility estimate of claim frequency for policyholder 3 is $0.56504676705 \times 0.00458085203848 + 0.43495323295 \times 0.00494560018799 = 0.00473950042532$.