ACSC/STAT 4703, Actuarial Models II
Fall 2020
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Homework Sheet 7
Model Solutions

Basic Questions

1. *An insurance company has the following data on its policies:*

Policy limit	*Losses Limited to*			
	20,000	*50,000*	*100,000*	*500,000*
20,000	*1,800,712*			
50,000	*9,744,913*	*11,144,757*		
100,000	*21,397,938*	*36,895,869*	*37,078,835*	
500,000	*16,783,656*	*18,797,737*	*18,915,855*	*20,046,074*

Use this data to calculate the ILF from $20,000 to $500,000 using

(a) The direct ILF estimate.

The direct ILF from $20,000 to $500,000 is $\frac{20046074}{16783656} = 1.19438065222$.

(b) The incremental method.

The ILF from $20,000 to $50,000 is $\frac{11144757+36895869+18797737}{9744913+21397938+16783656} = 1.3946011755$

The ILF from $50,000 to $100,000 is $\frac{37078835+18915855}{36895869+18797737} = 1.00540607839$

The ILF from $100,000 to $500,000 is $\frac{20046074}{18915855} = 1.05974982363$

Therefore the ILF from $20,000 to $500,000 is $1.3946011755 \times 1.00540607839 \times 1.05974982363 = 1.48591814628$.

2. *For a certain line of insurance, the loss amount per claim follows a Weibull distribution with parameters $\tau = 2$ and θ. If the policy has a deductible per loss set at 0.2θ and a policy limit set at 3θ, by how much will the expected payment per loss increase if there is inflation of 6%?*

The expected payment on the policy is

$$\int_{0.2\theta}^{3\theta} e^{-\left(\frac{x}{\theta}\right)^2}\,dx = \frac{\theta}{\sqrt{2}}\int_{0.2\sqrt{2}}^{3\sqrt{2}} e^{-\frac{u^2}{2}}\,du = \frac{\theta}{\sqrt{2}}\left(\Phi(3\sqrt{2}) - \Phi(0.2\sqrt{2})\right) = 0.2748083\theta$$

After inflation of 6%, the loss amount follows a Weibull distribution with parameters $\tau = 2$ and 1.06θ. The expected payment is

$$\int_{0.2\theta}^{3\theta} e^{-\left(\frac{x}{1.06\theta}\right)^2}\,dx = \frac{1.06\theta}{\sqrt{2}}\int_{\frac{0.2\sqrt{2}}{1.06}}^{\frac{3\sqrt{2}}{1.06}} e^{-\frac{u^2}{2}}\,du = \frac{\theta}{\sqrt{2}}\left(\Phi\left(\frac{3\sqrt{2}}{1.06}\right) - \Phi\left(\frac{0.2\sqrt{2}}{1.06}\right)\right) = 0.27914429\theta$$

so the expected payment per loss increases by a factor of $\frac{0.2791429}{0.2748083} = 1.01577317716$.

3. *An insurance company charges a risk charge equal to the square of the average loss amount, divided by 50,000. It has the following data on a set of 800 claims from policies with limit $1,000,000.*

Losses Limited to	*20,000*	*50,000*	*100,000*	*500,000*	*1,000,000*
Total claimed	*4,030,561*	*9,075,070*	*16,189,091*	*43,178,156*	*51,263,984*

Calculate the ILF from $50,000 to $1,000,000.

For a policy limit of $50,000 the average loss amount is $\frac{9075070}{800} = 11343.8375$. The premium with the risk charge is therefore $11343.8375 + \frac{11343.8375^2}{50000} = 13917.4904845$.

For a policy limit of $1000,000 the average loss amount is $\frac{51263984}{800} = 64079.98$. The premium with the risk charge is therefore $64079.98 + \frac{64079.98^2}{50000} = 146204.856736$. Therefore the ILF is $\frac{146204.856736}{13917.4904845} = 10.5051163425$.

Standard Questions

4. *An insurer calculates the ILF from $500,000 to $1,000,000 on a particular policy is 1.103. The average loss per unit of exposure with the policy limit of $1,000,000 is $2,047. The insurer's premium also includes a risk charge equal to the square of the expected loss divided by 10,000. A reinsurer is willing to provide excess-of-loss reinsurance of $500,000 over $500,000 (that is, the attachment point is $500,000 and the limit on the reinsurer's payment is $500,000) with a loading of 20%.*

(a) Calculate the average loss per unit of exposure for a policy with limit $500,000.

With the risk charge, the insurer's premium for one unit of exposure with limit $1,000,000 is $2047 + \frac{2047^2}{10000} = 2466.0209$. Therefore, the insurer's premium for a policy with limit $500,000 is $\frac{2466.0209}{1.103} = 2235.73970988$. Now the expected loss per unit of exposure for a policy with limit $500,000 is the solution to $x + \frac{x^2}{10000} = 2235.73970988$ which is $x = \sqrt{5000^2 + 22357397.0988} - 5000 = 1881.67109784$.

(b) Calculate the premium the insurance company should charge for a policy with limit $1,000,000 if they buy excess-of-loss reinsurance.

The expected payment on the reinsurance policy per unit of exposure is $2047 - 1881.67109784 = 165.32890216$. With a loading of 20%, the reinsurance premium is $1.20 \times 165.32890216 = 198.394682592$. Therefore if the insurer buys reinsurance, their premium would be $2235.73970988 + 198.394682592 = \$2,434.13$.

5. *An insurer sells a policy with limit \$1,000,000 with the premium equal to the expected payment plus a risk charge equal to the square of expected loss divided by 10,000. It calculates a trend factor of 1.047 for expected payments on this policy. A reinsurer offers excess-of-loss reinsurance of \$500,000 over \$500,000 for a 20% loading on the expected reinsurance payment. The trend factor for expected payments on a policy with limit \$500,000 is 1.044. The insurer finds that buying reinsurance would not affect its premium before applying trend factors. After applying trend factors, buying reinsurance allows the insurer to lower its premium by 0.5%. What is the expected payment on the policy with limit \$1,000,000 before trend factors are applied.*

   Since buying reinsurance would not change the premium, let x be the expected payment with a policy limit of \$1,000,000 and let y be the expected payment with a policy limit of \$500,000. The premium without reinsurance is $x+\frac{x^2}{10000}$, while the premium with reinsurance is $y+\frac{y^2}{10000}+1.2(x-y)$. Since these are equal, we get

$$\begin{aligned} x-y+\frac{x^2-y^2}{10000} &= 1.2(x-y)\\ \frac{(x+y)(x-y)}{10000} &= 0.2(x-y)\\ x+y &= 2000 \end{aligned}$$

   After applying the trend factors, we have that the expected payment on the policy with limit \$1,000,000 is $1.047x$, and the expected payment on the policy with limit \$500,000 is $1.044y$. The new premium without reinsurance is therefore $1.047x+\frac{1.047^2x^2}{10000}$, and the new premium with reinsurance is $1.044y+\frac{1.044^2y^2}{10000}+1.2(1.047x-1.044y)$. Since this is 4.5% lower than the premium without insurance, we get

$$\begin{aligned} 0.995\left(1.047x+\frac{1.047^2x^2}{10000}\right) &= 1.044y+\frac{1.044^2y^2}{10000}+1.2(1.047x-1.044y)\\ \frac{1.090727955x^2-1.089936y^2}{10000} &= 0.214635x-0.2088y\\ 1.089936(x-y)(x+y)+0.000791955x^2 &= 4234.35x-2088(x+y)\\ 2179.872(x-y)+0.000791955x^2 &= 4234.35x-4176000\\ 0.000791955x^2+125.394x-183744 &= 0\\ x &= \frac{-125.394+\sqrt{125.394^2+4\times 0.000791955\times 183744}}{2\times 0.000791955} = 1452 \end{aligned}$$