

ACSC/STAT 4703, Actuarial Models II

FALL 2021

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Homework Sheet 4

Due: Thursday 4th November: 11:30 AM

1. An insurance company sells tenant's insurance. It estimates that the standard deviation of the aggregate annual claim is \$82 and the mean is \$75.
 - (a) How many years history are needed for an individual or group to be assigned full credibility? (Use $r = 0.1$, $p = 0.95$.)

The standard net premium for this policy is \$75. An individual has claimed a total of \$833 in the last 3 years.
 - (b) What is the net Credibility premium for this individual, using limited fluctuation credibility?
2. A liability insurance company classifies companies as high, medium or low risk. Annual claims from high risk companies follow an inverse gamma distribution with $\alpha = 2.7$ and $\theta = 8000$. Annual claims from medium risk companies follow an inverse gamma distribution with $\alpha = 2.9$ and $\theta = 2500$. Annual claims from low risk companies follow a Pareto distribution with $\alpha = 3.3$ and $\theta = 1200$. 5% of companies are high risk, 40% are medium risk and 55% are low risk.
 - (a) Calculate the expectation and variance of the aggregate annual claims from a randomly chosen company.
 - (b) Given that a company's annual claims over the past 3 years are \$1300, \$13,440 and \$3,020, what are the expectation and variance of the company's claims next year?

Standard Questions

3. A group health insurance company has the following standards for full credibility: 480 person-years for frequency and 369 claims for severity.

The book estimates are 0.7 claims per person-year for claim frequency, and \$732 per claim for claim severity. Find the single standard in terms of person-years that gives the same premium for

 - (a) A company that has 304 claims from 291 person-years, with an average claim amount of \$308.
 - (b) A company that has 94 claims from 310 person-years, with an average claim amount of \$958.

4. A medical liability insurer classifies doctors as “low-risk” and “high-risk”. It estimates that 70% of doctors are low-risk. Annual claims from low-risk doctors are modelled as following a Pareto distribution with $\alpha = 3.8$ and $\theta = 1,340$. Annual claims from high-risk doctors have mean \$935 and variance 1,503,060.

It is considering modelling claims for high-risk doctors using either a Pareto distribution or an inverse gamma distribution, with parameters fitted using the method of moments. Which of these distributions would result in a higher Bayes premium for a doctor whose annual claims in the previous two years were \$146 and \$3,632?

5. An insurance company is pricing its policies for fire insurance. It insures 6 buildings, owned by 3 companies. Buildings 1, 2, 3 and 4 are in area A, while buildings 5 and 6 are in area B. Buildings 1, 2, and 5 are owned by company X, buildings 3 and 6 are owned by company Y, and building 4 is owned by company Z. The insurance company is setting a greatest accuracy credibility premium for each building. It will use a formula $\hat{X}_{i,2} = \beta_{i,0} + \sum_{j=1}^6 \alpha_{ij} X_{j,1}$ where $X_{i,k}$ is the total loss for building i in year j , to calculate the credibility premium. It makes the following modelling assumptions:

- The prior expected aggregate claims and variances of aggregate claims for each building are given in the following table:

Building	Expected aggregate claims	Variance of aggregate claims
1	\$ 4,904	483,600
2	\$ 2,048	301,500
3	\$ 3,360	289,700
4	\$11,421	1,004,200
5	\$ 9,310	1,143,000
6	\$ 5,082	821,100

- For a given building, the correlation between the losses in different years is 0.45. (Recall $\text{Corr}(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X) \text{Var}(Y)}}$)
- The correlation between aggregate claims for buildings in the same region in different years is 0.32 if owned by the same company, and 0.22 if owned by different companies.
- The correlation between aggregate claims for buildings owned by the same company in different regions is 0.25.
- Aggregate claims for buildings in different regions owned by different companies are independent.

Find a set of equations which can determine the values of $\beta_{1,0}$, and α_{1j} . [You do not need to solve these equations.]