ACSC/STAT 4703, Actuarial Models II

FALL 2021
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Homework Sheet 3

Model Solutions

Basic Questions

1. *A homeowner's house is valued at \$840,000, but is insured at \$360,000. The insurer requires 70% coverage for full insurance. The home sustains \$12,600 from fire. The policy has a deductible of \$5,000, which decreases linearly to zero when the total cost of the loss is \$15,000. How much does the insurance company reimburse?*

   70% of \$840,000 is \$588,000, so the insurance pays $\frac{360000}{588000} = 61.22\%$ of claims. The deductible is $\frac{15000-12600}{10000} \times 5000 = \$1,200$. The insurance therefore pays $\frac{360}{588}(12600 - 1200) = \$6,979.59$.

2. *An inland marine insurance company has two lines of coverage with different expected loss ratios, and has the following data on recent claims:*

Policy Type	*Policy Year*	*Earned Premiums*	*Expected Loss Ratio*	*Losses paid to date*
	2018	*\$4,200,000*	*0.78*	*\$3,200,000*
Train	*2019*	*\$4,600,000*	*0.77*	*\$2,900,000*
	2020	*\$6,500,000*	*0.78*	*\$4,800,000*
	2018	*\$6,600,000*	*0.74*	*\$3,200,000*
Truck	*2019*	*\$7,700,000*	*0.75*	*\$2,250,000*
	2020	*\$9,300,000*	*0.74*	*\$2,150,000*

Calculate the loss reserves at the end of 2020.

We calculate the expected losses and the expected unpaid losses.

Policy Type	Policy Year	Expected total Losses	Losses paid to date	Reserves Needed
	2018	\$3,276,000	\$3,200,000	\$76,000
Train	2019	\$3,542,000	\$2,900,000	\$642,000
	2020	\$5,070,000	\$4,800,000	\$270,000
	2018	\$4,884,000	\$3,200,000	\$1,684,000
Truck	2019	\$5,775,000	\$2,250,000	\$3,525,000
	2020	\$6,882,000	\$2,150,000	\$4,732,000
Total				\$ 10,929,000

So the total loss reserves needed at the end of 2020 are $10,929,000.

3. *The following table shows the paid losses on claims from one line of business of an insurance company over the past 5 years.*

Accident year	*Earned premiums*	*Development year 0*	*1*	*2*	*3*	*4*
2016	*6990*	*3347*	*1052*	*327*	*532*	*285*
2017	*5473*	*2863*	*2096*	*188*	*525*	
2018	*11117*	*4331*	*2671*	*869*		
2019	*11931*	*3797*	*2101*			
2020	*15229*	*4542*				

Assume that all payments on claims arising from accidents in 2016 have now been settled. Estimate the future payments arising each year from open claims arising from accidents in each calendar year using

(a) The loss development triangle method

First we compute the cumulative losses

Accident year	Development year 0	1	2	3	4
2016	3347	4399	4726	5258	5543
2017	2863	4959	5147	5672	
2018	4331	7002	7871		
2019	3797	5898			
2020	4542				

Now we compute the loss development factors:

Mean

0/1	$\frac{22258}{14338} = 1.55237829544$
1/2	$\frac{17744}{16360} = 1.08459657702$
2/3	$\frac{10930}{9873} = 1.10705965765$
3/4	$\frac{5543}{5258} = 1.05420311906$

Using these values to complete the table gives the following cumulative losses:

Accident year	Development year 0	1	2	3	4	
LDF		1.55237829544	1.08459657702	1.10705965765	1.05420311906	
2017					5672	5979.440
2018				7871	8713.667	9185.974
2019			5898	6396.951	7081.806	7465.662
2020		4542	7050.902	7647.384	8466.111	8925.000

The future payments are the differences between consecutive years:

Accident	Development year				
year	0	1	2	3	4
2017				307	
2018			843	472	
2019		499	685	384	
2020	2509	596	819	459	

Average

The loss development factors are:

0/1	$\frac{1}{4}\left(\frac{4399}{3347}+\frac{4959}{2863}+\frac{7002}{4331}+\frac{5898}{3797}\right)=1.55411469785$
1/2	$\frac{1}{3}\left(\frac{4726}{4399}+\frac{5147}{4959}+\frac{7871}{7002}\right)=1.07878444772$
2/3	$\frac{1}{2}\left(\frac{5258}{4726}+\frac{5672}{5147}\right)=1.10728496712$
3/4	$\frac{5543}{5258}=1.05420311906$

Using these values to complete the table gives the following cumulative losses:

Accident	Development year				
year	0	1	2	3	4
LDF		1.55411469785	1.07878444772	1.10728496712	1.05420311906
2017				5672	5979.440
2018			7871	8715.440	9187.844
2019		5898	6362.671	7045.290	7427.166
2020	4542	7058.789	7614.912	8431.877	8888.911

The future payments are the differences between consecutive years:

Accident	Development year				
year	0	1	2	3	4
2017					307.4401
2018				844.4400	472.4040
2019			464.6707	682.6189	381.8767
2020		2516.789	556.1228	816.9656	457.0340

(b) The Bornhuetter-Ferguson method with expected loss ratio 0.81.

Using the mean LDFs from part (a), we get the following:

Development	Cumulative proportion of losses paid		Proportion of losses paid	
Year	mean LDF	average LDF	mean LDF	average LDF
0	0.5089075	0.5109737	0.5089075	0.5109737
1	0.7900170	0.7941118	0.28110948	0.28313804
2	0.8568498	0.8566754	0.06683274	0.06256366
3	0.9485838	0.9485838	0.09173404	0.09190839
4	1.0000000	1.0000000	0.05141620	0.05141620

This gives the following reserves for mean LDF:

Accident year	Earned premiums	Expected Total claims	Development year 0	1	2	3	4
2017	5473	4433.13					227.9347
2018	11117	9004.77				826.0439	462.9911
2019	11931	9664.11			645.8789	886.5279	496.8918
2020	15229	12335.49		3467.623	824.4145	1131.5844	634.2440

and the following reserves for average LDF:

Accident year	Earned premiums	Expected Total payments	Development year 0	1	2	3	4
2017	5473	4433.13					227.9347
2018	11117	9004.77				827.6139	462.9911
2019	11931	9664.11			604.6221	888.2128	496.8918
2020	15229	12335.49		3492.646	771.7533	1133.7351	634.2440

4. *An actuary is reviewing the following claims data:*

No. of closed claims

Acc. Year	*Development Year 0*	*1*	*2*	*3*	*4*	*Ult.*
2016	*1075*	*2723*	*3298*	*3619*	*3666*	*3721*
2017	*2392*	*4337*	*5035*	*5418*		*5535*
2018	*4570*	*7042*	*8513*			*9311*
2019	*4197*	*8827*				*11945*
2020	*3107*					*6769*

Total paid losses on closed claims (000's)

Acc. Year	*Development Year 0*	*1*	*2*	*3*	*4*
2016	*2424*	*10146*	*10048*	*12217*	*15284*
2017	*5653*	*12384*	*19361*	*18659*	
2018	*10942*	*22642*	*25571*		
2019	*11111*	*24353*			
2020	*5983*				

(a) Calculate tables of percentage of claims closed and cumulative average losses.

For percentages of claims closed, we divide the claims closed by the ultimate claims closed:

Acc. Year	Development Year 0	1	2	3	4
2016	28.9	73.2	88.6	97.3	98.5
2017	43.2	78.4	91.0	97.9	
2018	49.1	75.6	91.4		
2019	35.1	73.9			
2020	45.9				

For cumulative average losses, we just divide the second table by the first.

Acc. Year	Development Year 0	1	2	3	4
2016	2,255	3,726	3,047	3,376	4,169
2017	2,363	2,855	3,845	3,444	
2018	2,394	3,215	3,004		
2019	2,647	2,759			
2020	1,926				

(b) Adjust the total loss table to use the current disposal rate.

We multiply the aggregate cumulative losses by the current disposal rate divided by the original disposal rate.

Acc.	Development Year				
Year	0	1	2	3	4
2016	3851	10246	10365	12296	15284
2017	6004	11679	19460	18659	
2018	10233	22123	25571		
2019	14515	24353			
2020	5983				

(c) Use the chain ladder method, with mean loss development factors to estimate claim development based on the adjusted numbers. Compare this to the chain ladder method on aggregate payments on closed claims.

The mean loss development factors are:

Development	LDF	
Year	Adjusted	Original
0/1	$\frac{68401}{34603} = 1.97673612115$	$\frac{69525}{30130} = 2.30750082974$
1/2	$\frac{55396}{44048} = 1.25762804214$	$\frac{54980}{45172} = 1.21712565306$
2/3	$\frac{30955}{29825} = 1.03788767812$	$\frac{30876}{29409} = 1.04988268897$
3/4	$\frac{15284}{12296} = 1.24300585556$	$\frac{15284}{12217} = 1.25104362773$

Using these values, we estimate the following cumulative losses:

Acc.	Development Year				
Year	0	1	2	3	4
2016	3851.240	10245.52	10365.13	12295.81	15284.00
2017	6004.159	11679.28	19459.53	18659.00	23193.25
2018	10232.779	22122.90	25571.00	26539.83	32989.16
2019	14515.001	24353.00	30627.02	31787.40	39511.93
2020	5983.000	11826.81	14873.73	15437.26	19188.61

Acc.	Development Year				
Year	0	1	2	3	4
2016	2424	10146.00	10048.00	12217.00	15284.00
2017	5653	12384.00	19361.00	18659.00	23343.22
2018	10942	22642.00	25571.00	26846.55	33586.21
2019	11111	24353.00	29640.66	31119.22	38931.50
2020	5983	13805.78	16803.37	17641.56	22070.36

Thus the reserves are:

Acc.	Development Year				
Year	0	1	2	3	4
2017					4534
2018				969	6449
2019			6274	1160	7725
2020		5844	3047	564	3751

Acc.	Development Year				
Year	0	1	2	3	4
2017					4684
2018				1276	6740
2019			5288	1479	7812
2020		7823	2998	838	4429

Standard Questions

5. *An insurance company has the following aggregate loss development data:*

Accident	Earned	Development year				
year	*premiums*	*0*	*1*	*2*	*3*	*4*
2016	*80929*	*12628*	*23111*	*39897*	*54644*	*58812*
2017	*80863*	*14270*	*26105*	*45201*	*61893*	
2018	*80874*	*14693*	*26991*	*46577*		
2019	*66143*	*13435*	*24570*			
2020	*91734*	*17247*				

(a) Use this data to estimate the loss development factors using the average method, and use both the chain ladder method and the Bornhuetter-Fergusson method with expected loss ratio 0.83 to estimate reserves for 2021.

The average loss development factors are given by

0/1	$\frac{1}{4}\left(\frac{23111}{12628}+\frac{26105}{14270}+\frac{26991}{14693}+\frac{24570}{13435}\right)=1.83132606001$
1/2	$\frac{1}{3}\left(\frac{39897}{23111}+\frac{45201}{26105}+\frac{46577}{26991}\right)=1.72782582139$
2/3	$\frac{1}{2}\left(\frac{54644}{39897}+\frac{61893}{45201}\right)=1.36945532719$
3/4	$\frac{58812}{54644}=1.07627552888$

This gives us the expected cumulative loss table

Accident	Development year				
year	0	1	2	3	4
2017				61893	66613.92
2018			46577	63785.12	68650.36
2019		24570	42452.68	58137.05	62571.48
2020	17247	31584.88	54573.17	74735.52	80436.01

and the reserves

Accident	Development year				
year	0	1	2	3	4
2017					4721
2018				17208	4865
2019			17883	15684	4434
2020		14338	22988	20162	5700

Thus, the expected reserve payments in 2021 are $4721+17208+17883+14338=\$54,150$.

For the Bornhuetter-Fergusson method, the cumulative proportion of losses paid is given by

Development Year	Cumulative proportion of losses paid	Proportion of losses paid
0	0.2144189	0.21441888
1	0.3926709	0.17825201
2	0.6784669	0.28579601
3	0.9291301	0.25066321
4	1.0000000	0.07086989

Thus, the expected loss payments are given in the following table:

Accident year	Earned premiums	Expected Total claims	Development year 0	1	2	3	4
2017	80863	67116					4757
2018	80874	67125				16826	4757
2019	66143	54899			15690	13761	3891
2020	91734	76139		13572	21760	19085	5396

so the total loss reserve for 2021 is $13572+15690+16826+4757 = \$50,845$.

(b) How much would the loss reserves be changed if the losses for accident year 2017, development year 3 were increased by \$20,000?

The average loss development factors with this increase are given by

0/1	$\frac{1}{4}\left(\frac{23111}{12628}+\frac{26105}{14270}+\frac{26991}{14693}+\frac{24570}{13435}\right)=1.83132606001$
1/2	$\frac{1}{3}\left(\frac{39897}{23111}+\frac{45201}{26105}+\frac{46577}{26991}\right)=1.72782582139$
2/3	$\frac{1}{2}\left(\frac{54644}{39897}+\frac{81893}{45201}\right)=1.59068937069$
3/4	$\frac{58812}{54644}=1.07627552888$

This gives us the expected cumulative loss table

Accident year	Development year 0	1	2	3	4
2017				81893	88139.43
2018			46577	74089.54	79740.76
2019		24570	42452.68	67529.03	72679.84
2020	17247	31584.88	54573.17	86808.96	93430.36

which gives the following reserves

Accident year	Development year 0	1	2	3	4
2017					6246
2018				27513	5651
2019			17883	25076	5151
2020		14338	22988	32236	6621

Thus, the expected reserve payments in 2021 are $6246+27513+17883+14338 = \$65,980$.

For the Bornhuetter-Fergusson method, the cumulative proportion of losses paid is given by

Development Year	Cumulative proportion of losses paid	Proportion of losses paid
0	0.1845974	0.18459737
1	0.3380580	0.15346061
2	0.5841053	0.24604733
3	0.9291301	0.34502480
4	1.0000000	0.07086989

Thus, the expected loss payments are given in the following table:

Accident year	Earned premiums	Expected Total claims	Development year 0	1	2	3	4
2017	80863	67116					4757
2018	80874	67125				23160	4757
2019	66143	54899			13508	18941	3891
2020	91734	76139		11684	18734	26270	5396

so the total loss reserve for 2021 is $11684+13508+23160+4757 = \$53,109$.