ACSC/STAT 4703, Actuarial Models II

FALL 2022
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Homework Sheet 4

Due: Thursday 3rd November: 17:30

1. An insurance company sells health insurance. It estimates that the standard deviation of the aggregate annual claim is \$32 and the mean is \$195.

   (a) How many years history are needed for an individual or group to be assigned full credibility? (Use $r = 0.1$, $p = 0.9$.)

   The standard net premium for this policy is \$195. An individual has claimed a total of \$133 in the last 4 years.

   (b) What is the net Credibility premium for this individual, using limited fluctuation credibility?

2. A fire insurance company classifies companies as high, medium or low risk. Annual claims from high risk companies follow a Pareto distribution with $\alpha = 4.7$ and $\theta = 12000$. Annual claims from medium risk companies follow a Weibull distribution with $\tau = 0.6$ and $\theta = 1500$. Annual claims from low risk companies follow a gamma distribution with $\alpha = 0.7$ and $\theta = 2000$. 15% of companies are high risk, 60% are medium risk and 25% are low risk.

   (a) Calculate the expectation and variance of the aggregate annual claims from a randomly chosen company.

   (b) Given that a company's annual claims over the past 3 years are \$700, \$1,440 and \$320, what are the expectation and variance of the company's claims next year?

Standard Questions

3. A home insurance company sets the standard for full credibility as 622 house-years. The book estimates are 0.07 claims per house-year for claim frequency and \$4,321 per claim for claim severity.

   The company changes the standard to 540 house-years for frequency and 86 claims for severity. For one policyholder with 11 person-years of history, this change results in the annual premium reducing from \$510 to \$449.11. How many claims did this policyholder make during the last 11 years?

4. An automobile insurer classifies drivers as "low-risk" and "high-risk". It estimates that 85% of drivers are low-risk. Annual claims from low-risk

drivers are modelled as following a Weibull distribution with $\tau = 0.6$ and $\theta = 385$ [mean 579.2616, variance 1037098]. Annual claims from high-risk drivers have mean \$1205 and variance 1,830,400.

It uses a Bayesian premium for each driver. For a particular driver with one year's experience, the net premium when modelling claims for high-risk drivers as following a gamma distribution (with $\alpha = 0.7932829$ and $\theta = 1519.004$ to match the given mean and variance) is \$100 more than when modelling claims for high-risk drivers as following an inverse gamma distribution (with $\alpha = 2.7932829$ and $\theta = 2160.906$ to match the given mean and variance). What were this driver's aggregate claims for the year?

5. An insurance company is pricing a professional liability insurance policy for a company. It has 5 years of past history for this company, and the annual claims from year i are denoted X_i. It uses the formula $\hat{X}_6 = \alpha_0 + \sum_{i=1}^{5} \alpha_i X_i$. It makes the following assumptions about the losses each year:

   - The expected aggregate claims was \$3052 in Year 1 and has been increasing by 4% inflation each year since then.
   - The coefficient of variation for aggregate claims is 2.7 in each year.
   - The correlation between losses in years i and j is $0.84(0.93^{|i-j|})$ if $i, j \neq 3$ and $0.62(0.93^{|i-3|})$ if $j = 3$. The change in Year 3 is to cover parental leave for one of the consultants working for the company. (Recall $\mathrm{Corr}(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)}}$)

   Find a set of equations which can determine the values of α_i for $i = 0, 1, \ldots, 5$. [You do not need to solve these equations.]