ACSC/STAT 4703, Actuarial Models II

FALL 2022
Toby Kenney

Homework Sheet 7

Model Solutions

Basic Questions

1. *An insurance company has the following data on its policies:*

Policy limit	*Losses Limited to*				
	50,000	*100,000*	*200,000*	*500,000*	*1,000,000*
50,000	*2,295,020*				
100,000	*6,405,601*	*6,962,250*			
200,000	*9,036,806*	*10,339,041*	*10,744,125*		
500,000	*14,832,105*	*16,246,821*	*17,383,225*	*18,641,393*	
1,000,000	*10,390,552*	*11,537,920*	*12,346,002*	*13,780,532*	*14,016,403*

Use this data to calculate the ILF from $50,000 to $1,000,000 using

(a) The direct ILF estimate.

The direct ILF estimate is $\frac{14016403}{10390552} = 1.34895653282$

(b) The incremental method.

The incremental ILF is

$$\frac{45086032}{40665064} \times \frac{40473352}{38123782} \times \frac{32421925}{29729227} \times \frac{14016403}{13780532} = 1.30562820396$$

2. *For a certain line of insurance, the loss amount per claim follows a gamma distribution with parameters* $\alpha = 0.3$ *and* θ. *If the policy has a deductible per loss set at* 0.1θ *and a policy limit set at* 2.5θ *(for the current value of* θ*), by how much will the expected payment per loss increase if there is inflation of 7%?*

The expected policy payment per loss before inflation is

$$\begin{aligned}
&\theta \int_{0.1}^{2.6} (x-0.1)\frac{x^{-0.7}e^{-x}}{\Gamma(0.3)}\,dx + 2.5\theta\int_{2.6}^{\infty}\frac{x^{-0.7}e^{-x}}{\Gamma(0.3)}\,dx\\
=&\theta\left(\int_{0.1}^{2.6}\frac{x^{0.3}e^{-x}}{\Gamma(0.3)}\,dx - 0.1\int_{0.1}^{2.6}\frac{x^{-0.7}e^{-x}}{\Gamma(0.3)}\,dx\right) + 2.5\theta\int_{2.6}^{\infty}\frac{x^{-0.7}e^{-x}}{\Gamma(0.3)}\,dx\\
=&\theta\left(0.3\int_{0.1}^{2.6}\frac{x^{0.3}e^{-x}}{\Gamma(1.3)}\,dx - 0.1\int_{0.1}^{2.6}\frac{x^{-0.7}e^{-x}}{\Gamma(0.3)}\,dx\right) + 2.5\theta\int_{2.6}^{\infty}\frac{x^{-0.7}e^{-x}}{\Gamma(0.3)}\,dx\\
&= 0.23346628\theta
\end{aligned}$$

After inflation of 7%, the loss amount follows a Gamma distribution with parameters $\alpha = 0.3$ and 1.07θ. The expected policy payment per loss is

$$\begin{aligned}
&1.07\theta\int_{\frac{0.1}{1.07}}^{\frac{2.6}{1.07}}\left(x-\frac{0.1}{1.07}\right)\frac{x^{-0.7}e^{-x}}{\Gamma(0.3)}\,dx + 2.5\theta\int_{\frac{2.6}{1.07}}^{\infty}\frac{x^{-0.7}e^{-x}}{\Gamma(0.3)}\,dx\\
=&1.07\theta\left(\int_{\frac{0.1}{1.07}}^{\frac{2.6}{1.07}}\frac{x^{0.3}e^{-x}}{\Gamma(0.3)}\,dx - \frac{0.1}{1.07}\int_{\frac{0.1}{1.07}}^{\frac{2.6}{1.07}}\frac{x^{-0.7}e^{-x}}{\Gamma(0.3)}\,dx\right) + 2.5\theta\int_{\frac{2.6}{1.07}}^{\infty}\frac{x^{-0.7}e^{-x}}{\Gamma(0.3)}\,dx\\
=&1.07\theta\left(0.3\int_{\frac{0.1}{1.07}}^{\frac{2.6}{1.07}}\frac{x^{0.3}e^{-x}}{\Gamma(1.3)}\,dx - \frac{0.1}{1.07}\int_{\frac{0.1}{1.07}}^{\frac{2.6}{1.07}}\frac{x^{-0.7}e^{-x}}{\Gamma(0.3)}\,dx\right) + 2.5\theta\int_{\frac{2.6}{1.07}}^{\infty}\frac{x^{-0.7}e^{-x}}{\Gamma(0.3)}\,dx\\
&= 0.25089983\theta
\end{aligned}$$

Thus, the expected payment per loss increases by a factor $\frac{0.2508983}{0.2334628} = 1.07468213351$. This is a 7.47% increase.

3. *An insurance company charges a risk charge equal to the square of the average loss amount, divided by 100,000. It has the following data on a set of 1693 claims from policies with limit $1,000,000.*

Losses Limited to	*200,000*	*500,000*	*1,000,000*
Total claimed	*$93,543,632*	*$112,984,361*	*$126,093,821*

Calculate the ILF from $200,000 to $1,000,000.

The pure premium for limit \$200,000 is $\frac{93543632}{1693} = 55253.1789722$. The risk charge is $\frac{55253.1789722^2}{100000} = 30529.1378653$. Thus the total premium is $55253.1789722 + 30529.1378653 = 85782.3168375$. The pure premium for limit \$1,000,000 is $\frac{126093821}{1693} = 74479.5162434$. The risk charge is $\frac{74479.5162434^2}{100000} = 55471.9833985$, so the total premium is $74479.5162434 + 55471.9833985 = 129951.499642$. The ILF is therefore $\frac{129951.499642}{85782.3168375} = 1.51489845965$.

Standard Questions

4. *An insurer sets its premiums for an insurance contract with policy limit 500,000 or 1,000,000 as the expected payment plus a 10% loading, plus a risk charge equal to the square of the expected payment divided by 50,000. Using these premiums, the ILF from 500,000 to 1,000,000 is 1.45. A reinsurer offers reinsurance of 500,000 over 500,000 for a premium of $143. Using this reinsurance policy, the original insurer can reduce the ILF to 1.43. What is the reinsurer's loading on this policy?*

   Let the expected loss limited to $500,000 be a and the expected loss limited to $1,000,000 be b. The insurer's premiums for insurance with limit $500,000 and $1,000,000 are $P = 1.1a + \frac{a^2}{50000}$ and $Q = 1.1b + \frac{b^2}{50000}$ respectively. By buying reinsurance with a premium $143, the insurer's ILF is 1.43. Thus, we have

$$\begin{aligned} \frac{P+143}{P} &= 1.43 \\ P + 143 &= 1.43P \\ 0.43P &= 143 \\ P &= 332.558139535 \end{aligned}$$

   Since the ILF without reinsurance is 1.45, it follows that $Q = 1.45 \times 332.558139535 = 482.209302326$. Now we solve for a and b

$$\begin{aligned} 1.1a + \frac{a^2}{50000} &= 332.558139535 \\ a^2 + 55000a - 16627906.9768 &= 0 \\ a &= \frac{\sqrt{55000^2 + 4 \times 16627906.9768} - 55000}{2} \\ &= 300.6817718 \end{aligned}$$

$$\begin{aligned} 1.1b + \frac{b^2}{50000} &= 482.209302326 \\ b^2 + 55000b - 24110465.1163 &= 0 \\ b &= \frac{\sqrt{55000^2 + 4 \times 24110465.1163} - 55000}{2} \\ &= 434.93270295 \end{aligned}$$

   Thus, the expected payment on the reinsurance policy is $b - a = 134.25093115$ so the reinsurer's loading is $\frac{143}{134.25093115} - 1 = 6.52\%$.

5. *An insurer sells policies with limits \$1,000,000 and \$2,000,000. The trend factor for losses limited to \$1,000,000 is 1.052. The trend factor for losses limited to \$2,000,000 is 1.044. The insurer's loading for policies with limit \$1,000,000 is 25%. For policies with limit \$2,000,000, the insurer buys reinsurance from a reinsurer. The ILF from \$1,000,000 to \$2,000,000 decreases from 1.36 in 2021 to 1.35 in 2022. What is the reinsurer's loading on this reinsurance.*

   Let a_{2021} and a_{2022} be the expected losses with limit \$1,000,000 in 2021 and 2022 respectively. Let b_{2021} and b_{2022} be the expected losses with limit \$2,000,000 in 2021 and 2022 respectively. In 2021 the premium for a policy with limit \$1,000,000 is $1.25a_{2021}$. Thus, the premium for a policy with limit \$2,000,000 is $1.36 \times 1.25a_{2021} = 1.7a_{2021}$. Hence the premium for the reinsurance is $(1.7 - 1.25)a_{2021} = 0.45a_{2021}$. If the loading is l, then we have $(1 + l)(b_{2021} - a_{2021}) = 0.45a_{2021}$.

   Similarly, the premium for reinsurance in 2022 is $(1.35 \times 1.25 - 1.25)a_{2022} = 0.4375a_{2022}$. From the trend factor, we have $a_{2022} = 1.052a_{2021}$, so the premium in 2022 is $0.4375 \times 1.052a_{2021} = 0.46025a_{2021}$. Thus $(1+l)(b_{2022} - 1.052a_{2021}) = 0.46025a_{2021}$. Substituting $b_{2022} = 1.044b_{2021}$ gives

$$(1 + l)(1.044b_{2021} - 1.052a_{2021}) = 0.46025a_{2021}$$

   Subtracting $1.044(1+l)(b_{2021} - a_{2021}) = 1.044 \times 0.45a_{2021}$ from both sides gives

$$(1 + l)(-0.008a_{2021}) = (0.46025 - 1.044 \times 0.45)a_{2021} = -0.00955a_{2021}$$

$$1 + l = \frac{0.00955}{0.008} = 1.19375$$

   so the loading is 19.38%.