ACSC/STAT 4703, Actuarial Models II

FALL 2023

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Homework Sheet 2

Due: Thursday 28th September: 14:30 AM

Basic Questions

1. An insurer models losses as following a distribution with distribution function $F(x) = 1-(1+x^4)^{-1}$. They find that $c_n = n^{\frac{1}{4}}$ and $d_n = n^{\frac{1}{4}}$ make the distribution of block maxima converge. What is the limiting distribution?

2. An insurer models losses as following a distribution with survival function $S(x) = (7x+cos(2\pi x))^{-1}$. What values of c_n and d_n make the distribution of block maxima converge, and what is the limiting distribution?

3. A loss follows a distribution from the MDA of a Fréchet distribution with $\xi = 0.4$. A reinsurer estimates that the probability of the loss exceeding \$500,000 is 0.006 and the probability of a loss exceeding \$1,000,000 is 0.002. What is the expected payment on an excess-of-loss reinsurance contract of \$1,000,000 over \$1,000,000 for this loss.

Standard Questions

4. The file `HW2_data.txt` contains 1,000,000 values of a random variable.

   (a) By dividing into blocks of different sizes, and using the `fit.GEV` function in the `QRM` package in `R`, estimate the tail index ξ.

   (b) The file `HW2_data.txt` contains 1,000,000 values of a random variable. Use the Hill estimator to estimate ξ at a range of different thresholds.

5. A insurer wants to calculate the ILF for a heavy-tailed loss. Based on previous data, they estimate that the distribution of the loss is in the MDA of a Fréchet distribution with $\xi = 2$. The ILF from \$500,000 to \$1,000,000 is 1.28 and the ILF from \$500,000 to \$2,000,000 is 1.76. Assuming the GPD approximation applies to losses above \$500,000, what is the ILF from \$500,000 to \$5,000,000?