

ACSC/STAT 4703, Actuarial Models II

FALL 2023

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Homework Sheet 5

Due: Thursday 2nd November: 14:30

1. A workers' compensation insurance company classifies companies as high, medium or low risk. Annual claims (in thousands) from high risk companies follow an inverse gamma distribution with $\alpha = 2.6$ and $\theta = 24$. Annual claims (in thousands) from medium risk companies follow a Pareto distribution with $\alpha = 8$ and $\theta = 42$. Annual claims (in thousands) from low risk companies follow a gamma distribution with $\alpha = 0.3$ and $\theta = 10$. 22% of companies are high risk, 48% are medium risk and 30% are low risk.
 - (a) Calculate the expectation and variance of the aggregate annual claims from a randomly chosen company.
 - (b) Given that a company's annual claims (in thousands) over the past 3 years were \$1.4, \$89.6 and \$1.2, what are the expectation and variance of the company's claims next year?
2. An insurance company sets the book pure premium for its homeowner's insurance at \$830. The expected process variance is 13,241,000 and the variance of hypothetical means is 291,000. If a policyholder has aggregate claims of \$11,400 over the past 19 years, calculate the credibility premium for this policyholder's next year's insurance using the Bühlmann model.
3. An insurance company has the following data on its group health insurance policy for a company.

Year	1	2	3	4	5
Exposure	5,021	7,425	7,591	7,309	7,393
Aggregate claims	\$1,084,200	\$1,909,400	\$2,996,200	\$972,800	\$1,664,000

The book premium is \$881 per unit of exposure. The variance of hypothetical means per unit of exposure is 24,951. The expected process variance per unit of exposure is 9,257,327,024. Using a Bühlmann-Straub model, calculate the credibility premium for Year 6 if the company has 8,815 units of exposure.

Standard Questions

4. An automobile insurer classifies drivers as "low-risk" and "high-risk". It estimates that 76% of drivers are low-risk. Annual claims from low-risk

drivers are modelled as following a gamma distribution with $\alpha = 1.6$ and $\theta = 585$. Annual claims from high-risk drivers are modelled as following an inverse gamma distribution with $\alpha = 6$ and $\theta = 6205$. A driver has two year's experience, and has claimed a total of \$1,514 in the past two years. Her net premium for the coming year, using the Bayesian approach is \$994. What were her two claims in the previous years? [You may need to use numerical methods to solve this.]

5. An insurance company uses the Bühlmann-Straub model to calculate credibility. A new customer pays the book premium for 304 units of exposure, paying a total net premium of \$79,290 in its first year. It claims a total of \$24,828. In the second year, there has been 5% inflation, the customer has 253 units of exposure and pays a credibility premium of \$65,021. The customer claims a total of \$31,090 in the second year. There is 4% inflation from the second year to the third year. In the third year, the customer has 370 units of exposure. What is the total premium they pay for these?
6. An insurance company is pricing a tenant's insurance policy for an individual. It has 4 years of past history for this individual, and the annual claims from year i are denoted X_i . It uses the formula $\hat{X}_5 = \alpha_0 + \sum_{i=1}^4 \alpha_i X_i$. It makes the following assumptions about the losses each year:
 - The expected aggregate claims was \$322 in Year 1 and has been increasing by 4% inflation each year since then.
 - The coefficient of variation for aggregate claims is 2.7 in each year.
 - The correlation between losses in years i and j is 0.47 if $i \neq j$. (Recall $\text{Corr}(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}}$)

Find a set of equations which can determine the values of α_i for $i = 0, 1, \dots, 5$. [You do not need to solve these equations.]