

ACSC/STAT 4720, Life Contingencies II  
Fall 2015

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Homework Sheet 7

Due: Friday 2nd October: 12:30 PM

## Basic Questions

1. An equity-linked insurance policy has the following properties:
  - Annual premiums are \$8,000.
  - Expense charges are 10% of the first premium and 1% of subsequent premiums.
  - There is a year-end management fee of 1% of fund value.
  - There is a year-end death benefit of 150% of fund value.
  - Surrenders receive full fund value.
  - GMMB is the total of the premiums paid.
  - The annual return is 7%.
  - The insurer's initial expenses are \$500 plus 40% of the first premium.
  - The insurer's renewal expenses are 0.5% of each subsequent premium.
  - Mortality is given by  $q_x = 0.0002 + 0.00001x$ .
  - The policy is sold to a life aged 39.
  - The policy matures in 10 years.
  - Surrenders happen at a rate of 3% per year.
  - (a) Calculate the projected fund value up to maturity of the policy.
  - (b) Calculate the profit signature of the policy.
  - (c) If the annual return is  $i = 0.01$ , what is the profit signature?
2. For an equity-linked insurance policy with the following properties:
  - Annual premiums are \$12,000.
  - Expense charges are 4% of the first premium and 1.5% of subsequent premiums.
  - There is a year-end management fee of 0.6% of fund value.
  - There is a year-end death benefit of 130% of fund value.
  - Surrenders receive full fund value.
  - GMMB is the total of the premiums paid.
  - The insurer's initial expenses are \$300 plus 20% of the first premium.
  - The insurer's renewal expenses are 0.3% of each subsequent premium.

- Mortality is given by  $q_x = 0.0002 + 0.00001x$ .
- The policy is sold to a life aged 43.
- The policy matures in 10 years.
- Surrenders happen at a rate of 2% per year.

(a) Use the following random numbers from a uniform distribution to simulate 10 years of annual returns following a log-normal distribution with  $\mu = 0.03$  and  $\sigma = 0.12$ .

0.5596925 0.6855121 0.2115462 0.8018509 0.2102174 0.4298370 0.3168010 0.5395057 0.8082324  
0.7455756

- (b) Use the simulated returns to calculate the account values for the next 10 years.  
(c) Calculate the profit signature for the policy.

## Standard Questions

3. An equity-linked insurance policy has the following properties:

- Annual premiums are \$9,000.
- Expense charges are 5% of the first premium and 1% of subsequent premiums.
- There is a year-end management fee of 1.4% of fund value.
- There is a year-end death benefit of 140% of fund value.
- Surrenders receive full fund value.
- GMMB is 110% of the total of the premiums paid.
- The insurer's initial expenses are \$200 plus 10% of the first premium.
- The insurer's renewal expenses are 0.4% of each subsequent premium.
- Mortality is given by  $q_x = 0.0002 + 0.00002x$ .
- The policy is sold to a life aged 36.
- The policy matures in 10 years.
- Surrenders happen at a rate of 2% per year.
- Annual returns are log-normally distributed with  $\mu = 0.05$  and  $\sigma = 0.16$ .

Simulate 5000 sets of 10-years' worth of annual returns. [Please include your code with your answer.]

- (a) Calculate the expected NPV of the policy at a risk discount rate of 10%.  
(b) Calculate the value of the Management expense fee needed to ensure that the probability of a net loss (negative NPV) is at most 5%, and the expected NPV is at least \$500.
4. For the policy in the previous question, suppose the fund value at the beginning of year 9 (before premiums are received) is \$74,205. Use a simulation to calculate a 95% quantile reserve at the start of year 9, if the reserve makes an annual return of  $i = 0.02$ .