ACSC/STAT 4720, Life Contingencies II
Fall 2015
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Homework Sheet 6
Model Solutions

Basic Questions

1. *A policyholder aged 54 buys a 10-year type B universal life insurance policy. The additional death benefit is \$150,000. The policyholder pays a premium of \$4,200 at the start of each year. The lifetable for the policyholder is:*

x	l_x	d_x
54	10000.00	8.33
55	9991.67	8.90
56	9982.77	9.54
57	9973.23	10.24
58	9962.99	11.01
59	9951.99	11.85
60	9940.14	12.78
61	9927.36	13.80
62	9913.56	14.91
63	9898.65	16.14

The cost of insurance is based on 105% of mortality in the above table and $i = 0.03$. *Expense charges are 2% of the account value (after each premium is paid). Assume the credited interest rate is* $i = 0.05$.

(a) Calculate the projected account value for the next 10 years.

AV_{t-1}	P_t	EC_t	CoI_t	interest	AV_t
0.00	4200	84.00	127.38	199.43	4188.05
4188.05	4200	167.76	136.21	404.20	8488.29
8488.29	4200	253.77	146.13	614.42	12902.82
12902.82	4200	342.06	157.00	830.19	17433.94
17433.94	4200	432.68	168.98	1051.61	22083.90
22083.90	4200	525.68	182.08	1278.81	26854.95
26854.95	4200	621.10	196.60	1511.86	31749.12
31749.12	4200	718.98	212.56	1750.88	36768.45
36768.45	4200	819.37	229.98	1995.95	41915.05
41915.05	4200	922.30	249.33	2247.17	47190.60

(b) Suppose the insurer earns an interest rate, $i = 0.04$, *and mortality follows the above table, initial expenses are \$700 and renewal expenses are 0.5% of account value each year after the first. Suppose there are no surrenders. Calculate the profit margin of this policy at a risk discount rate of* $i = 0.12$.

AV_{t-1}	P_t	E_t	I_t	EDB_t	EAV_t	Pr_t
0	0	700.00				−700.00
0.00	4200	0.00	168.00	128.44	4184.57	55.00
4188.05	4200	41.94	333.84	141.17	8480.73	58.06
8488.29	4200	63.44	504.99	155.68	12890.49	83.68
12902.94	4200	85.51	680.70	171.91	17416.04	110.04
17434.36	4200	108.17	861.05	190.17	22059.49	137.14
22084.80	4200	131.42	1046.14	210.58	26822.97	165.02
26856.59	4200	155.28	1236.05	233.67	31708.30	193.69
31751.79	4200	179.76	1430.88	259.63	36717.34	223.18
36772.54	4200	204.86	1630.71	288.64	41852.01	253.50
41921.02	4200	230.61	1835.62	321.52	47113.65	284.68

The EPV at $i = 0.12$ is therefore

$$55.00\times1.000000(1.12)^{-1}+58.06\times0.999167(1.12)^{-2}+83.68\times0.999110(1.12)^{-3}+110.04\times0.999046(1.12)^{-4}+137.14\times0.998976($$

The EPV of the premiums received is

$$4200(1.000000(1.12)^{-1}+0.999167(1.12)^{-2}+0.999110(1.12)^{-3}+0.999046(1.12)^{-4}+0.998976(1.12)^{-5}+0.998899(1.12)^{-6}+0.99$$

The profit margin is therefore

$$\frac{43.21}{26483.87} = 0.001631729$$

2. *A life aged 42 buys a 10-year type A universal life insurance policy with death benefit \$350,000. The annual premium is \$5,900. Mortality is as shown in the following table:*

x	l_x	d_x
42	10000.00	7.60
43	9992.40	7.87
44	9984.54	8.17
45	9976.37	8.51
46	9967.86	8.89
47	9958.97	9.31
48	9949.66	9.79
49	9939.87	10.32
50	9929.56	10.91
51	9918.65	11.58

The credited interest rate is $i = 0.06$*. Cost of insurance is based on mortality in the above table and* $i = 0.04$*. Expense charges are 2% of account value.*

(a) Project the account value for the next 10 years.

AV_{t-1}	P_t	EC_t	CoI_t	interest	AV_t
0.00	5900	118.00	251.49	331.83	5862.35
5862.35	5900	235.25	256.01	676.27	11947.35
11947.35	5900	356.95	261.01	1033.76	18263.16
18263.16	5900	483.26	266.72	1404.79	24817.97
24817.97	5900	614.36	273.03	1789.83	31620.42
31620.42	5900	750.41	279.84	2189.41	38679.58
38679.58	5900	891.59	287.61	2604.02	46004.40
46004.40	5900	1038.09	295.89	3034.22	53604.64
53604.64	5900	1190.09	304.81	3480.58	61490.33
61490.33	5900	1347.81	314.70	3943.67	69671.49

(b) Assume that the insurance company earns interest $i = 0.075$*; Mortality is 105% of the mortality in the lifetable. Initial expenses are $3,750; renewal expenses are 2% of premiums paid. The surrender charges and surrender rates are:*

Year	*Charge*	*rate*
1	*$4,200*	*4%*
2	*$3,500*	*5%*
3	*$3,200*	*4%*
4	*$2,800*	*3%*
5	*$2,200*	*3%*
6	*$1,400*	*4%*
7	*$900*	*4%*
8	*$400*	*5%*
9	*$0*	*7%*
10	*$0*	*100%*

Which of the following is the internal rate of return of the policy:

(i) $i = 0.0944$

(ii) $i = 0.1218$

(iii) $i = 0.1524$

(iv) $i = 0.1760$

AV_{t-1}	P_t	E_t	I_t	EDB_t	ESB_t	EAV_t	Pr_t
0	0	3750.00				−3750.00	
0.00	5900	0	442.50	279.30	66.44	5623.36	373.39
5862.35	5900	118	873.33	289.44	422.02	11340.60	465.62
11947.35	5900	118	1329.70	300.71	602.01	17517.57	638.76
18263.16	5900	118	1803.39	313.48	659.95	24051.87	823.25
24817.97	5900	118	2295.00	327.76	881.79	30643.08	1042.34
31620.42	5900	118	2805.18	343.55	1489.72	37095.95	1278.38
38679.58	5900	118	3334.62	361.60	1802.31	44118.60	1513.69
46004.40	5900	118	3883.98	381.55	2657.33	50868.89	1762.60
53604.64	5900	118	4454.00	403.79	4299.36	57120.03	2017.46
61490.33	5900	118	5045.42	429.06	69586.08	0.00	2302.62

We calculate the profit signature:

Year	Pr_t	P(in force)	Π_t
1	373.39	1.0000000	373.39
2	465.62	0.9592339	446.64
3	638.76	0.9105186	581.60
4	823.25	0.8733469	718.98
5	1042.34	0.8463877	882.22
6	1278.38	0.8202272	1048.56
7	1513.69	0.7866452	1190.74
8	1762.60	0.7543992	1329.70
9	2017.46	0.7158980	1444.30
10	2302.62	0.6650170	1531.28

(i)

$$373.39(1.0944)^{-1} + 446.64(1.0944)^{-2} + 581.60(1.0944)^{-3} + 718.98(1.0944)^{-4} + 882.22(1.0944)^{-5} + 1048.56(1.0944)^{-6} + 1190.74(1.0944)^{-7} + 1329.70(1.0944)^{-8} + 1444.30(1.0944)^{-9} + 1531.28(1.0944)^{-10} - 3750 =$$

(ii)

$$373.39(1.1218)^{-1} + 446.64(1.1218)^{-2} + 581.60(1.1218)^{-3} + 718.98(1.1218)^{-4} + 882.22(1.1218)^{-5} + 1048.56(1.1218)^{-6} + 1190.74(1.1218)^{-7} + 1329.70(1.1218)^{-8} + 1444.30(1.1218)^{-9} + 1531.28(1.1218)^{-10} - 3750 =$$

(iii)

$$373.39(1.1524)^{-1} + 446.64(1.1524)^{-2} + 581.60(1.1524)^{-3} + 718.98(1.1524)^{-4} + 882.22(1.1524)^{-5} + 1048.56(1.1524)^{-6} + 1190.74(1.1524)^{-7} + 1329.70(1.1524)^{-8} + 1444.30(1.1524)^{-9} + 1531.28(1.1524)^{-10} - 3750 =$$

(iv)

$$373.39(1.1760)^{-1} + 446.64(1.1760)^{-2} + 581.60(1.1760)^{-3} + 718.98(1.1760)^{-4} + 882.22(1.1760)^{-5} + 1048.56(1.1760)^{-6} + 1190.74(1.1760)^{-7} + 1329.70(1.1760)^{-8} + 1444.30(1.1760)^{-9} + 1531.28(1.1760)^{-10} - 3750 =$$

So (iv) $i = 0.1760$ is closest to the internal rate of return.
[After correcting an error, the correct IRR should be
$i = 0.1643$, which gives

$$373.39(1.1643)^{-1} + 446.64(1.1643)^{-2} + 581.60(1.1643)^{-3} + 718.98(1.1643)^{-4} + 882.22(1.1643)^{-5} + 1048.56(1.1643)^{-6} + 1190.74(1.1643)^{-7} + 1329.70(1.1643)^{-8} + 1444.30(1.1643)^{-9} + 1531.28(1.1643)^{-10} - 3750 =$$

]

3. *A life aged 39 has an annual type A Universal life insurance policy that has been in effect for 4 years.*

- *The current account value is $32,418.*
- *The annual premium is $6,300.*
- *The expense charge is 1.5% of account value.*
- *The credited interest rate is* $i = 0.04$.
- *The total death benefit is $100,000.*
- *The corridor factor requirement is 2.6.*
- *The insurance is priced using mortality rate* $q_{39} = 0.000192$ *and interest* $i = 0.03$.

Calculate the cost of insurance charge for the year.

If the CoI is C, then the account value at the end of the year is

$$1.04(0.985(32418 + 6300) - C) = 39662.72 - 1.04C$$

Under the corridor factor requirement, the minimum total death benefit is

$$2.6(39662.72 - 1.04C) = 103123.07 - 2.704C$$

This means that if $C > \frac{3123.07}{2.704} = 1154.98$ then the corridor factor does not applies and the CoI is

$$\begin{aligned}C &= 0.000192(100000 - (39662.72 - 1.04C))1.03^{-1} = 11.2473 + 0.0001938641C\\ 0.9998061359C &= 11.2473\\ C &= \frac{11.2473}{0.9998061359} = 11.25\end{aligned}$$

However, this is less than 1154.98, so the corridor factor applies, and the cost of insurance is

$$\begin{aligned}C &= 0.000192(1.6(39662.72 - 1.04C))1.03^{-1} = 11.8295 - 0.0003101825\\ 1.0003101825C &= 11.8295\\ C &= \frac{11.8295}{1.0003101825} = 11.83\end{aligned}$$

So the cost of insurance is $11.83.

Standard Questions

4. *Consider an annual type B universal life insurance policy with annual premiums of \$3,000, additional death benefit \$80,000 with no corridor factor requirement. The expense charge is 1.5% of account value.*

   Surrender charges and rates are

Year	*Charge*	*rate*
1	*\$2,200*	*4%*
2	*\$1,300*	*5%*
3	*\$800*	*4%*
4	*\$300*	*3%*
5	*\$0*	*100%*

   Initial expenses are \$800, and renewal expenses are \$30. Cost of insurance is based on mortality $q_x = 0.000402$ *and* $i = 0.04$. *The insurance company makes an annual rate of return equal to prime+1% (where the prime rate is set each year by the central bank). It offers credited interest as prime + a for some a. Calculate the value of a so that whatever the value of prime, the insurance company's profit margin on the policy at a risk discount rate of 10% is at least 5%. [Assume prime is always in the range 0–10%.]*

The cost of insurance is $0.000402 \times 80000(1.04)^{-1} = 30.92308$. If the account value at the start of the year is A_{t-1}, then the value after the premium, expenses and cost of insurance is $0.985(A_{t-1} + 3000) - 30.92308$, and the value the insurance company has to invest is $A_{t-1} + 2970$, so if the prime rate is i, then the account value at the end of the year is $A_t = (0.985(A_{t-1} + 3000) - 30.92308)(1 + i + a)$, while the insurance company has $(A_{t-1} + 2970)(1 + i + 0.01)$. The expected death benefit is $0.000402(A_t + 80000)$. The expected surrender benefit is $0.999598 s_t(A_t - SC_t)$. The emerging surplus is therefore

$$
\begin{aligned}
&(A_{t-1} + 2970)(1 + i + 0.01) - 0.000402(A_t + 80000) - 0.999598 s_t(A_t - SC_t) - 0.999598(1 - s_t)A_t \\
&= (A_{t-1} + 2970)(1 + i + 0.01) - A_t - 32.16 + 0.999598 s_t SC_t \\
&= (A_{t-1} + 2970)(1 + i + 0.01) - (0.985(A_{t-1} + 3000) - 30.92308)(1 + i + a) - 32.16 + 0.999598 s_t SC_t \\
&= A_{t-1}(1 + i + 0.01 - 0.985(1 + i + a)) + 43.46308 + 45.92308i - 2924.077a + 0.999598 s_t SC_t \\
&= A_{t-1}(0.015i + 0.025 - 0.985a) + 43.46308 + 45.92308i - 2924.077a + 0.999598 s_t SC_t
\end{aligned}
$$

From this, we see that the emerging surplus in each year is an increasing function of i. Also, increasing the prime rate in previous years will increase the account value, which will increase future emerging surplusses, provided $0.985a < 0.025 + 0.015i$. Therefore the smallest emerging surplus is if $i = 0$.

We therefore calculate the account values and emerging surplusses based on $i = 0$ in each year.

The account values are then

$$
\begin{aligned}
A_t &= (0.985(A_{t-1} + 3000) - 30.92308)(1 + a) \\
&= (0.985A_{t-1} + 2924.07692)(1 + a)
\end{aligned}
$$

This gives

$$
\begin{aligned}
A_1 &= (2924.07692)(1+a) \\
A_2 &= (0.985(2924.07692)(1+a) + 2924.07692)(1+a) \\
&\dots \\
A_n &= 2924.07692(1+a)\frac{1-0.985^n(1+a)^n}{0.015-0.985a}
\end{aligned}
$$

The emerging surplus is then

$$\left(2924.07692(1+a)\frac{1-0.985^n(1+a)^n}{0.015-0.985a}\right)(0.025-0.985a) + 43.46308 - 2924.077a + 0.999598s_tSC_t$$

And the profit vector is

$${}_{n-1}p_x\left(\left(2924.07692(1+a)\frac{1-0.985^n(1+a)^n}{0.015-0.985a}\right)(0.025-0.985a) + 43.46308 - 2924.077a + 0.999598s_tSC_t\right)$$

At a risk discount rate of 10%, the EPV of the policy is

$$\sum_{n=1}^{5} {}_{n-1}p_x\left(\left(2924.07692(1+a)\frac{1-0.985^n(1+a)^n}{0.015-0.985a}\right)(0.025-0.985a) + 43.46308 - 2924.077a + 0.999598s_tSC_t\right)(1.1)^{-n} - 800$$

The EPV of the premiums is

$$
\begin{aligned}
&\sum_{n=0}^{4} {}_np_x 3000(1.1)^{-n} \\
&= 3000\left(1 + \frac{0.96\times 0.999598}{1.1} + \frac{0.95\times 0.96\times 0.999598^2}{1.1^2} + \frac{0.95\times 0.96^2\times 0.999598^3}{1.1^2} + \frac{0.95\times 0.96^2\times 0.97\times 0.999598^4}{1.1^4}\right) \\
&= 11584.82
\end{aligned}
$$

To obtain a profit margin of 5%, the EPV of the policy needs to be $0.05\times 11584.82 = 579.241$.
We therefore need to solve

$$\sum_{n=1}^{5} {}_{n-1}p_x\left(\left(2924.07692(1+a)\frac{1-0.985^n(1+a)^n}{0.015-0.985a}\right)(0.025-0.985a) + 43.46308 - 2924.077a + 0.999598s_nSC_n\right)(1.1)^{-n} = 1379$$

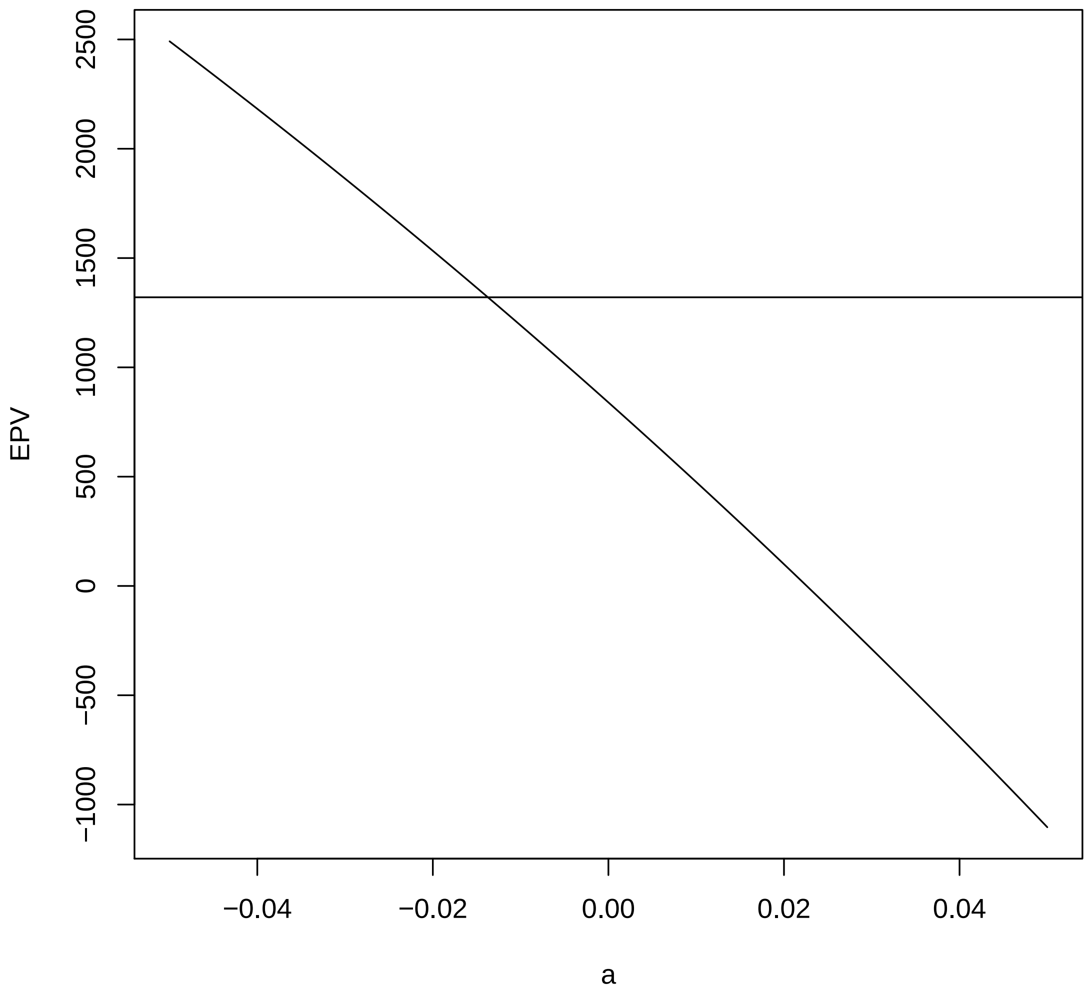
We also have that

$$
\begin{aligned}
&\sum_{n=1}^{5}(0.999598 s_{n\,n-1}p_x SC_n)1.1^{-n}\\
&= \frac{0.999598\times 0.04\times 2200}{1.1}+\frac{0.999598^2\times 0.96\times 0.05\times 1300}{1.1^2}+\frac{0.999598^3\times 0.96\times 0.95\times 0.04\times 800}{1.1}+\\
&\qquad \frac{0.999598^4\times 0.95\times 0.96^2\times 0.03\times 300}{1.1^4}\\
&= 158.770
\end{aligned}
$$

Which gives

$$
\sum_{n=1}^{5} {}_{n-1}p_x\left(\left(2924.07692(1+a)\frac{1-0.985^n(1+a)^n}{0.015-0.985a}\right)(0.025-0.985a)+43.46308-2924.077a\right)(1.1)^{-n}=1320.471
$$

Numerically evaluating this for large numbers of values of a, we see that $a=-0.01372$ achieves the desired profit margin.



2500
2000
1500
1000
500
0
−500
−1000
EPV
−0.04
−0.02
0.00
0.02
0.04
a