ACSC/STAT 4720, Life Contingencies II
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Homework Sheet 7
Model Solutions

Basic Questions

1. *An equity-linked insurance policy has the following properties:*

   - *Annual premiums are \$8,000.*
   - *Expense charges are 10% of the first premium and 1% of subsequent premiums.*
   - *There is a year-end management fee of 1% of fund value.*
   - *There is a year-end death benefit of 150% of fund value.*
   - *Surrenders receive full fund value.*
   - *GMMB is the total of the premiums paid.*
   - *The annual return is 7%.*
   - *The insurer's initial expenses are \$500 plus 40% of the first premium.*
   - *The insurer's renewal expenses are 0.5% of each subsequent premium.*
   - *Mortality is given by* $q_x = 0.0002 + 0.00001x$.
   - *The policy is sold to a life aged 39.*
   - *The policy matures in 10 years.*
   - *Surrenders happen at a rate of 3% per year.*

   (a) Calculate the projected fund value up to maturity of the policy.

t	Alloc. Prem.	Start Value	Int.	Fund before	Mgmt. Charge	Fund
1	7200	0.00	504.00	7704.00	77.04	7626.96
2	7920	7626.96	1088.29	16635.25	166.35	16468.89
3	7920	16468.89	1707.22	26096.12	260.96	25835.16
4	7920	25835.16	2362.86	36118.02	361.18	35756.84
5	7920	35756.84	3057.38	46734.22	467.34	46266.87
6	7920	46266.87	3793.08	57979.95	579.80	57400.15
7	7920	57400.15	4572.41	69892.57	698.93	69193.64
8	7920	69193.64	5397.96	82511.59	825.12	81686.48
9	7920	81686.48	6272.45	95878.93	958.79	94920.14
10	7920	94920.14	7198.81	110038.95	1100.39	108938.56

   (b) Calculate the profit signature of the policy.

t	Unalloc. Prem.	Expenses	Int.	Mgmt. Charge	Exp. Death Benefit	Pr_t
0	0	3700				−3700
1	800	0	56.00	77.04	2.25	930.79
2	80	40	2.80	166.35	4.94	204.21
3	80	40	2.80	260.96	7.88	295.88
4	80	40	2.80	361.18	11.08	392.90
5	80	40	2.80	467.34	14.57	495.57
6	80	40	2.80	579.80	18.37	604.23
7	80	40	2.80	698.93	22.49	719.24
8	80	40	2.80	825.12	26.96	840.96
9	80	40	2.80	958.79	31.80	969.79
10	80	40	2.80	1100.39	37.04	1106.15

t	P(in force)	Pr_t	Π_t
0	1.0000000	−3700.00	−3700.00
1	1.0000000	930.79	930.79
2	0.9694277	204.21	197.97
3	0.9397807	295.88	278.06
4	0.9110312	392.90	357.94
5	0.8831523	495.57	437.66
6	0.8561181	604.23	517.29
7	0.8299031	719.24	596.90
8	0.8044827	840.96	676.54
9	0.7798332	969.79	756.28
10	0.7559314	1106.15	836.17

(c) If the annual return is $i = 0.01$, what is the profit signature?

We project the fund value

t	Alloc. Prem.	Start Value	Int.	Fund before	Mgmt. Charge	Fund
1	0.00	7200	72.00	7272.00	72.72	7199.28
2	7199.28	7920	151.19	15270.47	152.70	15117.77
3	15117.77	7920	230.38	23268.15	232.68	23035.46
4	23035.46	7920	309.55	31265.02	312.65	30952.37
5	30952.37	7920	388.72	39261.09	392.61	38868.48
6	38868.48	7920	467.88	47256.37	472.56	46783.80
7	46783.80	7920	547.04	55250.84	552.51	54698.33
8	54698.33	7920	626.18	63244.52	632.45	62612.07
9	62612.07	7920	705.32	71237.39	712.37	70525.02
10	70525.02	7920	784.45	79229.47	792.29	78437.17

Calculate the profit vector

t	Unalloc. Prem.	Expenses	Int.	Mgmt. Charge	Exp. Death Benefit	GMMB Payment	Pr_t
0	0	3700				−3700	
1	800	0	8.0	72.72	2.12		878.60
2	80	40	0.4	152.70	4.54		188.57
3	80	40	0.4	232.68	7.03		266.06
4	80	40	0.4	312.65	9.60		343.45
5	80	40	0.4	392.61	12.24		420.77
6	80	40	0.4	472.56	14.97		497.99
7	80	40	0.4	552.51	17.78		575.13
8	80	40	0.4	632.45	20.66		652.18
9	80	40	0.4	712.37	23.63		729.15
10	80	40	0.4	792.29	26.67	1562.83	−756.80

Then the profit signature:

t	P(in force)	Pr_t	Π_t
0	1.0000000	−3700	−3700
1	1.0000000	878.60	878.60
2	0.9694277	188.57	182.80
3	0.9397807	266.06	250.03
4	0.9110312	343.45	312.90
5	0.8831523	420.77	371.60
6	0.8561181	497.99	426.34
7	0.8299031	575.13	477.30
8	0.8044827	652.18	524.67
9	0.7798332	729.15	568.61
10	0.7559314	−756.80	−572.09

2. *For an equity-linked insurance policy with the following properties:*

   - *Annual premiums are \$12,000.*
   - *Expense charges are 4% of the first premium and 1.5% of subsequent premiums.*
   - *There is a year-end management fee of 0.6% of fund value.*
   - *There is a year-end death benefit of 130% of fund value.*
   - *Surrenders receive full fund value.*
   - *GMMB is the total of the premiums paid.*
   - *The insurer's initial expenses are \$300 plus 20% of the first premium.*
   - *The insurer's renewal expenses are 0.3% of each subsequent premium.*
   - *Mortality is given by* $q_x = 0.0002 + 0.00001x$.
   - *The policy is sold to a life aged 43.*
   - *The policy matures in 10 years.*
   - *Surrenders happen at a rate of 2% per year.*

   (a) Use the following random numbers from a uniform distribution to simulate 10 years of annual returns following a log-normal distribution with $\mu = 0.03$ *and* $\sigma = 0.12$.

0.5596925 0.6855121 0.2115462 0.8018509 0.2102174 0.4298370 0.3168010 0.5395057 0.8082324 0.7455756

Year	U	$\Phi^{-1}(U)$	$e^{0.03+0.12\Phi^{-1}(U)}$
1	0.5596925	0.15018964	1.0491945
2	0.6855121	0.48316892	1.0919666
3	0.2115462	−0.80106779	0.9360109
4	0.8018509	0.84825100	1.1408688
5	0.2102174	−0.80566714	0.9354945
6	0.4298370	−0.17678917	1.0088240
7	0.3168010	−0.47666316	0.9731670
8	0.5395057	0.09918851	1.0427929
9	0.8082324	0.87140107	1.1440426
10	0.7455756	0.66063129	1.1154699

(b) Use the simulated returns to calculate the account values for the next 10 years.

t	Rate of Return	Alloc. Prem.	Start Value	Int.	Fund before	Mgmt. Charge	Fund Value
1	0.049194531	11520	0.00	566.72	12086.72	72.52	12014.20
2	0.091966578	11820	12014.20	2191.95	26026.15	156.16	25869.99
3	−0.063989079	11820	25869.99	−2411.75	35278.25	211.67	35066.58
4	0.140868849	11820	35066.58	6604.86	53491.43	320.95	53170.49
5	−0.064505542	11820	53170.49	−4192.25	60798.24	364.79	60433.45
6	0.008824004	11820	60433.45	637.56	72891.01	437.35	72453.67
7	−0.026833002	11820	72453.67	−2261.32	82012.35	492.07	81520.28
8	0.042792927	11820	81520.28	3994.30	97334.58	584.01	96750.57
9	0.144042599	11820	96750.57	15638.79	124209.36	745.26	123464.11
10	0.115469904	11820	123464.11	15621.24	150905.35	905.43	149999.92

(c) Calculate the profit signature for the policy.

Calculate the profit vector

t	Unalloc. Prem.	Expenses	Int.	Mgmt. Charge	Exp. Death Benefit	Pr_t
0	0	2700				−2700
1	480	0	23.61	72.52	2.27	573.86
2	180	36	13.24	156.16	4.97	308.43
3	180	36	-9.21	211.67	6.84	339.62
4	180	36	20.29	320.95	10.53	474.71
5	180	36	-9.29	364.79	12.15	487.35
6	180	36	1.27	437.35	14.78	567.84
7	180	36	-3.86	492.07	16.87	615.34
8	180	36	6.16	584.01	20.32	713.85
9	180	36	20.74	745.26	26.30	883.70
10	180	36	16.63	905.43	32.40	1033.66

Then the profit signature:

t	P(in force)	Pr_t	Π_t
0	1.0000000	−2700.00	−2700.00
1	1.0000000	573.86	573.86
2	0.9793826	308.43	302.07
3	0.9591807	339.62	325.75
4	0.9393861	474.71	445.93
5	0.9199908	487.35	448.36
6	0.9009869	567.84	511.61
7	0.8823667	615.34	542.95
8	0.8641227	713.85	616.86
9	0.8462475	883.70	747.83
10	0.8287337	1033.66	856.63

Standard Questions

3. An equity-linked insurance policy has the following properties:

- *Annual premiums are \$9,000.*
- *Expense charges are 5% of the first premium and 1% of subsequent premiums.*
- *There is a year-end management fee of 1.4% of fund value.*
- *There is a year-end death benefit of 140% of fund value.*
- *Surrenders receive full fund value.*
- *GMMB is 110% of the total of the premiums paid.*
- *The insurer's initial expenses are \$200 plus 10% of the first premium.*
- *The insurer's renewal expenses are 0.4% of each subsequent premium.*
- *Mortality is given by* $q_x = 0.0002 + 0.00002x$.
- *The policy is sold to a life aged 36.*
- *The policy matures in 10 years.*
- *Surrenders happen at a rate of 2% per year.*
- *Annual returns are log-normally distributed with* $\mu = 0.05$ *and* $\sigma = 0.16$.

Simulate 5000 sets of 10-years' worth of annual returns. [Please include your code with your answer.]

(a) Calculate the expected NPV of the policy at a risk discount rate of 10%.

We simulate 5,000 NPVs:

R-code:

```
randomUniform<-runif(50000)
dim(randomUniform)<-c(5000,10)
Ret<-exp(qnorm(randomUniform)*0.16+0.05)
```

```
#Ret gives 5000 sets of ten years of annual returns

ManFee<-0.014

NPV<-rep(0,5000)

for(i in 1:5000){
#First project account values:
    Apr<-c(8550,rep(8910,9))#Allocated premium
    AV<-rep(0,10)#Account value
    MC<-rep(0,10)#Management charge
    FV<-Apr[1]*Ret[i,1]
    MC[1]<-FV*ManFee
    AV[1]<-FV-MC[1]
    for(j in 2:10){
        FV<-(Apr[j]+AV[j-1])*Ret[i,j]
        MC[j]<-FV*ManFee
        AV[j]<-FV-MC[j]
    }
    UP<-c(450,rep(90,9))#Unallocated premium
    Exp<-c(0,rep(36,9))
    EDB<-0.4*AV*(0.0002+0.00002*(36:45))
    if(AV[10]<99000){
        Prt[10]<-Prt[10]+(AV[10]-99000)*(0.9998-0.00002*(36:45))
    }#Apply GMMB
    Prt<-(UP-Exp)*ret[i,]+MC-EDB #Profit vector
    Pif<-cumprod(c(1,0.9998-0.00002*(36:44)))*0.98^(0:9) #Probability in force
                    #=survival probability*(not surrendered probability)
    NPV[i]<-sum(Pif*Prt/1.1^(1:10))-1100
};

mean(NPV)
```

My simulation gives a mean NPV of $1,268.06.

(b) Calculate the value of the Management expense fee needed to ensure that the probability of a net loss (negative NPV) is at most 5%, and the expected NPV is at least $500.

We modify the code for part (a) by changing the value of ManFee, and returning the proportion of simulations that resulted in a negative profit.

We then run the simulation with different levels of Management Fee:

Management Fee	Expected NPV	Probability of Loss
0.01	404.23	0.2670
0.02	2493.98	0.2490
0.03	4384.75	0.2288
0.04	5993.94	0.2014
0.05	7392.86	0.1728
0.06	8760.91	0.1528
0.07	9553.33	0.1310
0.08	10425.85	0.1224
0.09	11258.40	0.0940
0.10	11905.28	0.0812
0.11	12276.00	0.0670
0.12	12730.57	0.0552
0.13	13271.62	0.0396
0.14	13521.02	0.0336
0.15	14031.48	0.0264

From this we see that the management fee should be a little more than 12%. [This is based on no reserves, if we added reserves, there would still be a substantial probability of loss.]

[Further simulation suggests the true value is around 12.39%.]

4. *For the policy in the previous question, suppose the fund value at the beginning of year 9 (before premiums are received) is \$74,205. Use a simulation to calculate a 95% quantile reserve at the start of year 9, if the reserve makes an annual return of* $i = 0.02$.

```
randomUniform<-runif(10000)
dim(randomUniform)<-c(5000,2)
Ret<-exp(qnorm(randomUniform)*0.16+0.05)


#Ret gives 5000 sets of ten years of annual returns

ManFee<-0.014

NPV<-rep(0,5000)

for(i in 1:5000){
#First project account values:
    FV<-(8910+74205)*Ret[i,1]
    MC[1]<-FV*ManFee
    AV[1]<-FV-MC[1]
    FV<-(8910+AV[1])*Ret[i,2]
    MC[2]<-FV*ManFee
    AV[2]<-FV-MC[2]
    EDB<-0.4*AV*(0.0002+0.00002*(44:45))
    Prt<-54*Ret[i,]+MC-EDB #Profit vector
    if(AV[2]<99000){
        Prt[2]<-Prt[2]+(AV[2]-99000)*(0.9998-0.00002*45)
```

```
    }#Apply GMMB
    Pif<-c(1,(0.9998-0.00002*44)*0.98) #Probability in force
                    #=survival probability*(not surrendered probability)
    NPV[i]<-sum(Pif*Prt/1.02^(1:2))
};

print(quantile(NPV,0.95))
```

My simulation gives \$7,543.82. [Repeated running gives the range 7500–7560.]